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SOLUTION OF A TRANSVERSE PLANE BENDING PROBLEM OF A LAMINATED CANTILEVER BEAM UNDER THE ACTION OF A NORMAL UNIFORM LOAD

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The paper presents an exact direct solution of a transverse plane bending problem of a laminated cantilever beam of small width under the action of a uniformly distributed load under the absolutely hard contact between the layers. The solution has been constructed with the aid of linear elasticity theory equations: to take into account the heterogeneous structure of the laminated beam, piecewise constant functions of elastic characteristics, which have been described analytically by means of shifted Heaviside functions, have been introduced into the Hooke's law relationships. During the solution of problem, the normal stresses were expressed from static equations in terms of an unknown function of tangential stress distribution over the cross-section height. The use of the obtained expressions in the Hooke's law relationships and Cauchy equations for linear strains made it possible to establish relationships between displacements and tangential stresses. The Cauchy equation that remained for angle strains gave a defining integro-differential equation, from which differential equations for the unknown tangential stress function and for all unknown integration functions have been derived. The solution of the derived equations is possible for the entire packet of layers without considering an individual layer, the final relations for stresses, strains and displacements describing the stress-strain state of the entire packet of composite beam layers. The constructed solution satisfies the boundary conditions and the conditions of the absolutely hard contact of the layers and is exact if the load distribution corresponds to the determined stress distribution. Using this solution, we have carried out a theoretical study of the stress-strain state of a three-layer beam. The obtained relation allow one to predict the strength and stiffness of multilayer structural composite elements and to construct application solutions for other elastic bending problems of laminated beams.

Keywords: beam, composite, layer, bending, stress, strains, displacements, warping.

Introduction. [redacted]

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CONCLUSIONS

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