

ISSN 2664-9969

TECHNOLOGY AUDIT AND PRODUCTION RESERVES



№ 6/1 (86),
2025

Editorial board

Editor-in-Chief

Marcelo S. Nagano, Professor, University of Sao Paulo, Brazil

Deputy Editor-in-Chief

Igor Dudnikov, Professor, Poltava State Agrarian University, Ukraine

Executive Editor of Technical Sciences

Talal Yusaf, Professor, Central Queensland University, Australia

Executive Editor of Economic Sciences

Hrabrin Bachev, Professor, Institute of Agricultural Economics Sofia, Bulgaria

Assistant Editor

Oleksandra Bilovod, Associate Professor, Poltava State Agrarian University, Ukraine

Mechanical Engineering

Oleg Akimov, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Oleksandra Bilovod**, Associate Professor, Poltava State Agrarian University, Ukraine; **Igor Dudnikov**, Professor, Poltava State Agrarian University, Ukraine; **Oleksandr Hondliakh**, Professor, National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute", Ukraine; **Oleksii Goryk**, Professor, Poltava State Agrarian University, Ukraine; **Avraam A. Konstantinidis**, Associate Professor, Aristotle University of Thessaloniki, Greece; **Myroslav Kindrachuk**, Professor, National Aviation University, Corresponding Member of the National Academy of Sciences of Ukraine, Kyiv, Ukraine; **Alexander Permyakov**, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Yuri Petrakov**, Professor, National Technical University of Ukraine "Kyiv Polytechnic Institute", Ukraine; **Ravipudi Venkata Rao**, Professor, Sardar Vallabhbhai National Institute of Technology, Surat, India; **Parthasarathy Sampathkumaran**, Professor, Sambhram Institution of Technology, India; **Viktor Sheychenko**, Professor, Poltava State Agrarian University, Ukraine; **Iwan Susanto**, PhD, National Dong Hwa University, Indonesia; **Kong Fah Tee**, PhD, King Fahd University of Petroleum and Minerals, Kingdom of Saudi Arabia; **Fazil Veliev**, Professor, Azerbaijan State University of Economics, Azerbaijan; **Sergii Vodennikov**, Professor, Zaporizhzhia Polytechnic National University, Ukraine; **Ling Zhou**, Professor, National Research Center of Pumps, Jiangsu University, China

Energy

Müslüm Arıcı, Professor, Kocaeli University, Turkey; **Boris Borts**, Senior Researcher, National Science Center, Kharkiv Institute of Physics and Technology, Ukraine; **Valeriy Chigirinsky**, Professor, Rudny Industrial University, Republic of Kazakhstan; **Ahmed Kadhim Hussein**, Professor, University of Babylon, Iraq; **Volodymyr Kravchenko**, Professor, Odesa Polytechnic National University, Ukraine; **Iaroslav Liashok**, Professor, Donetsk National Technical University, Ukraine; **Michail Lubkov**, Professor, Poltava Gravimetric Observatory within S. I. Subbotin Institute of Geophysics of National Academy of Sciences of Ukraine; **Mihai Lungu**, Professor, University of Craiova, Romania; **Yanqing Niu**, Professor, Xi'an Jiaotong University, China; **Andrii Perecrest**, Professor, Kremenchuk Mykhailo Ostrohradskyi National University, Ukraine; **Volodymyr Potapov**, Professor, State Biotechnological University, Ukraine; **Valery Tuz**, Professor, National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute", Ukraine; **Oleg Sinchuk**, Professor, Kryvyi Rih National University, Ukraine; **Roman Volianskyi**, Associate Professor, National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute", Ukraine; **Talal Yusaf**, Professor, Central Queensland University, Australia

Information and Control Systems

Sara Antomarioni, Research Associate, Universita Politecnica delle Marche, Italy; **Zeki Ayag**, Professor, Piri Reis University, Turkey; **Andrii Biloshchytskyi**, Professor, Taras Shevchenko National University of Kyiv, Ukraine; Astana IT University, Republic of Kazakhstan; **Dmytro Bugayko**, Professor, State University "Kyiv Aviation Institute", Ukraine; **Sergey Bushuyev**, Professor, Kyiv National University of Construction and Architecture, Ukraine; **Morteza Daneshmand**, PhD, Norwegian Institute of Bioeconomy Research, Norway; **Anatoly Dovbysh**, Professor, Sumy State University, Ukraine; **Sergii Filonenko**, Professor, State University "Kyiv Aviation Institute", Ukraine; **Kainizhamal Iklassova**, Associate Professor, M. Kozybayev North Kazakhstan University, Republic of Kazakhstan; **Palle Jorgensen**, Professor, University of Iowa, United States; **Valeriy Lakhno**, Professor, National University of Life and Environmental Sciences of Ukraine; **Vasyl Lytvyn**, Professor, Lviv Polytechnic National University, Ukraine; **Lakshmi Narayan Mishra**, Assistant Professor, Vellore Institute of Technology, India; **Vishnu Narayan Mishra**, Professor, Indira Gandhi National Tribal University, Amarkantak, India; **Dmytro Mykhalevskyi**, Professor, Vinnytsia National Technical University, Ukraine; **Marcelo S. Nagano**, Professor, University of Sao Paulo, Brazil; **Ahmad H. Sabry**, PhD, Shatt Al-Arab University College, Basra, Iraq; **Valerii Samsonkin**, Professor, National Transport University, Kyiv, Ukraine; **Orest Serediuk**, Professor, Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine; **Oleksandr Sotnikov**, Professor, Ivan Kozhedub Kharkiv National Air Force University, Ukraine; **Vasyl Teslyuk**, Professor, Lviv Polytechnic National University, Ukraine; **Cemil Tunc**, Professor, Istanbul Medipol University, Turkey; **Oleksandr Turovsky**, Professor, State University of Information and Communication Technologies, Ukraine

Chemical Engineering

Katerina E. Aifantis, Associate Professor, University of Florida, United States; **Ulaganathan Arisekar**, PhD, Tamil Nadu Dr. J. Jayalithaa Fisheries University, Nagapattinam, India; **Gilmar Ferreira Batalha**, Associate Professor, University of Sao Paulo, Brazil; **Olena Bilyk**, Professor, National University of Food Technologies, Kyiv, Ukraine; **Olga Bliznjuk**, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Diego Elustondo**, Scion (New Zealand Forest Research Institute Limited), Rotorua, New Zealand; **Olena Fedorenko**, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Svitlana Gorobets**, Professor, National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute", Ukraine; **Natalia Kosulina**, Professor, State Biotechnological University, Ukraine; **Oleg Mandryk**, Professor, Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine; **Andrii Marynin**, Senior Researcher, National University of Food Technologies, Kyiv, Ukraine; **Maria Paska**, Professor, Lviv State University of Physical Culture named after I. Bobersky, Ukraine; **Sulaksana Permana**, Dr. Eng., Universitas Indonesia (Alumni), Indonesia; **Olesia Priss**, Professor, Dmytro Motornyi Tavria State Agrotechnological University, Ukraine; **Yana Redko**, Professor, Kyiv National University of Technologies and Design, Ukraine; **Vsevolod Sklabinskiy**, Professor, Sumy State University, Ukraine; **Valentin Sviderskiy**, Professor, National Technical University of Ukraine "Ihor Sikorsky Kyiv Polytechnic Institute", Ukraine; **Mostapha Tarfaoui**, Professor, Ecole Nationale Supérieure de Techniques Avancées Bretagne, France; **Sagdat Tazhibayeva**, Professor, Al-Farabi Kazakh National University, Republic of Kazakhstan; **Dinara Tlevlessova**, PhD, Kazakh Scientific Research Institute of Processing and Food Industry LLP, Republic of Kazakhstan; **Sergij Vambol**, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Seydi Yikmiş**, PhD, Tekirdag Namik Kemal University, Turkey

Economics

Kamarulzaman Ab. Aziz, Professor, Multimedia University, Cyberjaya, Selangor, Malaysia; **Hatem Altaee**, Professor, Cihan University of Slemani, Iraq; **Kateryna Andriushchenko**, Professor, Kyiv National Economic University named after Vadym Hetman, Ukraine; **Hrabrin Bachev**, Professor, Institute of Agricultural Economics Sofia, Bulgaria; **Tomasz Bernat**, Professor, University of Szczecin, Poland; **Ionel Bostan**, Professor, Stefan cel Mare University of Suceava, Romania; **Igor Britchenko**, Professor, University of the National Education Commission, Krakow, Poland; **Narendra Dalei**, Full Professor, Central University of Punjab, Bathinda, India; **Adriana Grigorescu**, Professor, National University of Political Studies and Public Administration, Correspondent Member, Academy of Romanian Scientists, Bucharest, Romania; **Waldemar Izdebski**, Professor, University of Warsaw, Poland; **György Kocziszky**, Professor, University of Miskolc, Hungary; **Rab Nawaz Lodhi**, Associate Professor, University of the Punjab, Lahore, Pakistan; **Tamara Lozynska**, Professor, Poltava State Agrarian University, Ukraine; **Petro Makarenko**, Professor, Poltava State Agrarian University, Ukraine; **Viacheslav Makedon**, Professor, Oles Honchar Dnipro National University, Ukraine; **Roberto Micera**, PhD, University of Basilicata, Italy; **Robert Nowacki**, Assistant Professor, VIZJA University in Warsaw, Poland; **Gbadebo Olusegun Abidemi Odularu**, Professor, Howard University, Washington, D.C., United States; **Svitlana Onyshchenko**, Professor, Odessa National Maritime University, Ukraine; **Iryna Otenko**, Professor, Simon Kuznets Kharkiv National University of Economics, Ukraine; **Petro Pererva**, Professor, National Technical University "Kharkiv Polytechnic Institute", Ukraine; **Zarina Poberezhna**, Professor, State University "Kyiv Aviation Institute", Ukraine; **Oleksandr Pushkar**, Professor, Simon Kuznets Kharkiv National University of Economics, Ukraine; **Volodymyr Pysarenko**, Professor, Poltava State Agrarian University, Ukraine; **Juergen Seitz**, Professor, Baden-Wuerttemberg Cooperative State University Heidenheim, Germany; **Mirosław Skibniewski**, Professor, University of Maryland, United States; **Yochanan Shachmurove**, Professor, The City College of New York, United States; **Marcin Waldemar Staniewski**, Adjunct Professor, VIZJA University in Warsaw, Poland; **Adriana Tiron-Tudor**, Professor, Babes-Bolyai University, Cluj-Napoca, Romania; **Galyna Zakharchyn**, Professor, Lviv Polytechnic National University, Ukraine; **Ruslana Zhovnovach**, Professor, Central Ukrainian National Technical University, Ukraine

International Abstracting and Indexing:

- ✓ Scopus
- ✓ EBSCO
- ✓ FSTA
- ✓ IDEAS
- ✓ WorldCat
- ✓ OpenAIRE
- ✓ Crossref
- ✓ ERIH PLUS
- ✓ BASE
- ✓ CNKI
- ✓ EconBiz
- ✓ EconPapers
- ✓ MIAR
- ✓ Scilit
- ✓ Neliti

Publisher

TECHNOLOGY CENTER PC*

Establishers

TECHNOLOGY CENTER PC*
Poltava State Agrarian University

Editorial

TECHNOLOGY CENTER GROUP LTD

Published since 2011

Media ID in the Register of Media Entities R30-01128

Certificated by

Order of the MES of Ukraine
No. 1346 from 10.10.2025 (Category A)

Editorial office and publishing house addresses

Shatilova dacha st., 4, Kharkiv, Ukraine, 61165
<https://tarp.net.ua/>, <https://journals.uran.ua/tarp/>

Tel.: +380 (57) 7508990

E-mail: tapr@entc.com.ua, tarp.nauka@gmail.com

Conventional printed sheet 10.75. Publisher's signature 10.00
Format 60×84 1/8

Редакційна колегія

Головний редактор

Marcelo S. Nagano, Professor, University of Sao Paulo, Brazil

Заступник головного редактора

Дудніков Ігор Анатолійович, к. т. н., професор, Полтавський державний аграрний університет

Відповідальний редактор з технічних наук

Talal Yusaf, Professor, Central Queensland University, Australia

Відповідальний редактор з економічних наук

Hrabrin Bachev, Professor, Institute of Agricultural Economics Sofia, Bulgaria

Відповідальний секретар

Біловод Олександра Іванівна, к. т. н., доцент, Полтавський державний аграрний університет

Машинобудування

Акімов Олег Вікторович, д. т. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Біловод Олександра Іванівна**, к. т. н., доц., Полтавський державний аграрний університет; **Дудніков Ігор Анатолійович**, к. т. н., проф., Полтавський державний аграрний університет; **Гондях Олександр Володимирович**, д. т. н., проф., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Горик Олексій Володимирович**, д. т. н., проф., Полтавський державний аграрний університет; **Avraam A. Konstantinidis**, Associate Professor, Aristotle University of Thessaloniki, Greece; **Кіндрачук Мирослав Васильович**, д. т. н., проф., Національний авіаційний університет, член-кореспондент НАН України, Київ; **Пермяков Олександр Анатолійович**, д. т. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Петраков Юрій Володимирович**, д. т. н., проф., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Ravipudi Venkata Rao**, Professor, Sardar Vallabhbhai National Institute of Technology, Surat, India; **Parthasarathy Sampathkumaran**, Professor, Sambhram Institution of Technology, India; **Шейченко Віктор Олександрович**, д. т. н., проф., Полтавський державний аграрний університет; **Iwan Susanto**, PhD, National Dong Hwa University, Indonesia; **Kong Fah Tee**, PhD, King Fahd University of Petroleum and Minerals, Kingdom of Saudi Arabia; **Fazil Veliev**, Professor, Azerbaijan State University of Economics, Azerbaijan; **Воденніков Сергій Анатолійович**, д. т. н., проф., Національний університет «Запорізька політехніка»; **Ling Zhou**, Professor, National Research Center of Pumps, Jiangsu University, China

Енергетика

Müslüm Arıcı, Professor, Kocaeli University, Turkey; **Борц Борис Вікторович**, д. т. н., стар. наук. спів., Національний науковий центр, Харківський фізико-технічний інститут НАН України; **Valeriy Chigirinsky**, Professor, Rudny Industrial University, Republic of Kazakhstan; **Ahmed Kadhim Hussein**, Professor, University of Babylon, Iraq; **Кравченко Володимир Петрович**, д. т. н., проф., Національний університет «Одеська політехніка»; **Ляшок Ярослав Олександрович**, д. е. н., проф., Донецький національний технічний університет; **Лубков Михайло Валерійович**, д. ф.-м. н., проф., Полтавська гравіметрична обсерваторія Інституту геофізики ім. С. І. Субботіна Національної академії наук України; **Mihai Lungu**, Professor, University of Craiova, Romania; **Yanqing Niu**, Professor, Xi'an Jiaotong University, China; **Перекрест Андрій Леонідович**, д. т. н., проф., Кременчуцький національний університет імені Михайла Остроградського; **Потапов Володимир Олексійович**, д. т. н., проф., Державний біотехнологічний університет; **Туз Валерій Омелянович**, д. т. н., проф., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Сінчук Олег Миколайович**, д. т. н., проф., ДВНЗ «Криворізький національний університет»; **Волянський Роман Сергійович**, к. т. н., доц., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Talal Yusaf**, Professor, Central Queensland University, Australia

Інформаційно-керуючі системи

Sara Antomarioni, Assistant Professor, Universita Politecnica delle Marche, Italy; **Zeki Ayag**, Professor, Piri Reis University, Turkey; **Andrii Biloshchytskyi**, Professor, Taras Shevchenko National University of Kyiv, Ukraine; Astana IT University, Kazakhstan; **Бурайко Дмитро Олександрович**, д. е. н., проф., Державний університет «Київський авіаційний інститут»; **Бушуєв Сергій Дмитрович**, д. т. н., проф., Київський національний університет будівництва і архітектури; **Morteza Daneshmand**, PhD, Norwegian Institute of Bioeconomy Research, Norway; **Довбиш Анатолій Степанович**, д. т. н., проф., Сумський державний університет; **Філоненко Сергій Федорович**, д. т. н., проф., Державний університет «Київський авіаційний інститут»; **Kainizhamal Iklassova**, Associate Professor, M. Kozybayev North Kazakhstan University, Republic of Kazakhstan; **Palle Jorgensen**, Professor, University of Iowa, United States; **Лахно Валерій Анатолійович**, д. т. н., проф., Національний університет біоресурсів і природокористування України; **Литвин Василь Володимирович**, д. т. н., проф., Національний університет «Львівська політехніка»; **Lakshmi Narayan Mishra**, Assistant Professor, Vellore Institute of Technology, India; **Vishnu Narayan Mishra**, Professor, Indira Gandhi National Tribal University, Amarkantak, India; **Михалевський Дмитро Валерійович**, д. т. н., проф., Вінницький національний технічний університет; **Marcelo S. Nagano**, Professor, University of Sao Paulo, Brazil; **Ahmad H. Sabry**, PhD, Shatt Al-Arab University College, Basra, Iraq; **Самсонкін Валерій Миколайович**, д. т. н., проф., Національний транспортний університет, Київ; **Середюк Орест Євгенович**, д. т. н., проф., Івано-Франківський національний технічний університет нафти і газу; **Сотніков Олександр Михайлович**, д. т. н., проф., Харківський національний університет Повітряних Сил імені Івана Кожедуба; **Теслюк Василь Миколайович**, д. т. н., проф., Національний університет «Львівська політехніка»; **Cemil Tunc**, Professor, Istanbul Medipol University, Turkey; **Туровський Олександр Леонідович**, д. т. н., проф., Державний університет інформаційно-комунікаційних технологій

Хімічна інженерія

Katerina E. Aifantis, Associate Professor, University of Florida, United States; **Ulaganathan Arisekar**, PhD, Tamil Nadu Dr. J. Jayalalithaa Fisheries University, Nagapattinam, India; **Gilmar Ferreira Batalha**, Associate Professor, University of Sao Paulo, Brazil; **Білик Олена Анатоліївна**, к. т. н., проф., Національний університет харчових технологій, Київ; **Близнюк Ольга Миколаївна**, д. т. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Diego Elustondo**, Scion (New Zealand Forest Research Institute Limited), Rotorua, New Zealand; **Федоренко Олена Юрївна**, д. т. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Горобець Світлана Василівна**, д. т. н., проф., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Косуліна Наталія Геннадіївна**, д. т. н., проф., Державний біотехнологічний університет; **Мандрик Олег Миколайович**, д. т. н., проф., Івано-Франківський національний технічний університет нафти і газу; **Маринін Андрій Іванович**, к. т. н., стар. наук. спів., Національний університет харчових технологій, Київ; **Паска Марія Зіновіївна**, д. в. н., проф., Львівський державний університет фізичної культури імені Івана Боберського; **Sulaksana Permana**, Dr. Eng., Universitas Indonesia (Alumni), Indonesia; **Прісс Олеся Петрівна**, д. т. н., проф., Таврійський державний агротехнологічний університет імені Дмитра Моторного; **Редько Яна Володимирівна**, д. т. н., проф., Київський національний університет технологій та дизайну; **Склабінський Всеволод Іванович**, д. т. н., проф., Сумський державний університет; **Свідерський Валентин Анатолійович**, д. т. н., проф., Національний технічний університет України «Київський політехнічний інститут ім. Ігоря Сікорського»; **Mostapha Tarfaoui**, Professor, Ecole Nationale Supérieure de Techniques Avancées Bretagne, France; **Sagdat Tazhibayeva**, Professor, Al-Farabi Kazakh National University, Republic of Kazakhstan; **Dinara Tlevlessova**, PhD, Kazakh Scientific Research Institute of Processing and Food Industry LLP, Republic of Kazakhstan; **Вамболь Сергій Олександрович**, д. т. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Seydi Yikmiş**, PhD, Tekirdag Namik Kemal University, Turkey

Економічні науки

Kamarulzaman Ab. Aziz, Professor, Multimedia University, Cyberjaya, Selangor, Malaysia; **Hatem Altaee**, Professor, Cihan University of Slemani, Iraq; **Андрющенко Катерина Анатоліївна**, д. е. н., проф., Київський національний економічний університет імені Вадима Гетьмана; **Hrabrin Bachev**, Professor, Institute of Agricultural Economics Sofia, Bulgaria; **Tomasz Bernat**, Professor, University of Szczecin, Poland; **Ionel Bostan**, Professor, Stefan cel Mare University of Suceava, Romania; **Igor Britchenko**, Professor, University of the National Education Commission, Krakow, Poland; **Narendra Dalei**, Full Professor, Central University of Punjab, Bathinda, India; **Adriana Grigorescu**, Professor, National University of Political Studies and Public Administration, Correspondent Member, Academy of Romanian Scientists, Bucharest, Romania; **Waldemar Izdebski**, Professor, University of Warsaw, Poland; **György Kocziszky**, Professor, University of Miskolc, Hungary; **Rab Nawaz Lodhi**, Associate Professor, University of the Punjab, Lahore, Pakistan; **Лозинська Тамара Миколаївна**, д. н. держ. упр., проф., Полтавський державний аграрний університет; **Макаренко Петро Миколайович**, д. е. н., проф., Полтавський державний аграрний університет; **Македон Вячеслав Владиславович**, д. е. н., проф., Дніпровський національний університет імені Олеся Гончара; **Roberto Micera**, PhD, University of Basilicata, Italy; **Robert Nowacki**, Assistant Professor, VIZJA University in Warsaw, Poland; **Gbadebo Olusegun Abidemi Odularu**, Professor, Howard University, Washington, D.C., United States; **Онищенко Світлана Петрівна**, д. е. н., проф., Одеський національний морський університет; **Отенко Ірина Павлівна**, д. е. н., проф., Харківський національний економічний університет ім. С. Кузнеця; **Перерва Петро Григорович**, д. е. н., проф., Національний технічний університет «Харківський політехнічний інститут»; **Побережна Заріна Миколаївна**, д. е. н., проф., Державний університет «Київський авіаційний інститут»; **Пушкар Олександр Іванович**, д. е. н., проф., Харківський національний економічний університет ім. Семена Кузнеця; **Писаренко Володимир Вікторович**, д. е. н., проф., Полтавський державний аграрний університет; **Juergen Seitz**, Professor, Baden-Wuerttemberg Cooperative State University Heidenheim, Germany; **Mirosław Skibniewski**, Professor, University of Maryland, United States; **Yochanan Shachmurove**, Professor, The City College of New York, United States; **Marcin Waldemar Staniewski**, Adjunct Professor, VIZJA University in Warsaw, Poland; **Adriana Tiron-Tudor**, Professor, Babes-Bolyai University, Cluj-Napoca, Romania; **Захарчин Галина МIRONІВНА**, д. е. н., проф., Національний університет «Львівська політехніка»; **Жовновач Руслана Іванівна**, д. е. н., проф., Центральноукраїнський національний технічний університет

Міжнародна представленість та індексація журналу:

- ✓ Scopus
- ✓ EBSCO
- ✓ FSTA
- ✓ IDEAS
- ✓ WorldCat
- ✓ OpenAIRE
- ✓ Crossref
- ✓ ERIH PLUS
- ✓ BASE
- ✓ CNKI
- ✓ EconBiz
- ✓ EconPapers
- ✓ MIAR
- ✓ Scilit
- ✓ Neliti

Видавець

ПП "ТЕХНОЛОГІЧНИЙ ЦЕНТР"

Засновники

ПП "ТЕХНОЛОГІЧНИЙ ЦЕНТР"
Полтавський державний аграрний університет

Редакція

ТОВ "ТЕХНОЛОГІЧНИЙ ЦЕНТР ГРУП"

Видається з 2011 року

Ідентифікатор медіа в Реєстрі суб'єктів у сфері медіа R30-01128

Атестовано

Наказом Міністерства освіти і науки України
№ 1346 від 10.10.2025 (Категорія А)

Адреса редакції та видавництва

вул. Шатилова дача, 4, м. Харків, Україна, 61165
<https://tarpr.net.ua/>, <https://journals.urau.ua/tarpr/>

Тел.: +380 (57) 7508990

E-mail: tarpr@entc.com.ua, tarpr.nauka@gmail.com

Ум. друк. арк. 10,75. Обл.-вид. арк. 10,00
Формат 60×84 1/8

TECHNOLOGY AUDIT AND PRODUCTION RESERVES

No. 6/1(86), 2025

CONTENTS



MECHANICAL ENGINEERING TECHNOLOGY

- 6 Development of special cold spray nozzle for internal surface coating deposition
Oleksandr Shorinov, Andrii Volkov, Nina Savchenko, Serhii Markovych, Kostiantyn Danko, Roman Ipatov
- 12 Design optimization of a mold for producing a complex-geometry blade
Kateryna Maiorova, Olga Shypul, Ihor Lysochenko, Taras Hoptar, Liudmyla Kapitanova, Iryna Voronko
- 20 Determination of the efficiency of shot blasting of metal surfaces
Oleksii Goryk, Oleksandr Brykun, Oleg Kalashnykov, Andrii Buchynskyi



MECHANICS

- 26 Analytical determination of absolute deformations and boundaries of the contact zone of a deformable wheel with a deformable surface
Volodymyr Kovbasa, Nataliia Priliepo
- 34 Evaluation of Prager and Chaboche models for reliable prediction of elastoplastic behaviour in low-cycle fatigue
Amira Aboussalih, Salah Hammoudi, Kamel Fedaoui, Karim Arar, Lazhar Baroura



METALLURGICAL TECHNOLOGY

- 43 Determination of conditions for preventing chemical wear of induction furnace lining in the "cupola furnace – induction furnace" duplex process
Denis Nikolaev, Vadym Selivorstov, Yuriy Dotsenko, Iryna Osypenko, Eugene Kuznetsow



TECHNOLOGY AND SYSTEM OF POWER SUPPLY

- 48 Development of a method for modeling the magnetic state and assessing the electromechanical characteristics of a vortex layer of ferromagnetic particles moving in a rotating magnetic field
Henrikh Polishchikov, Pavlo Zhukov
- 57 Optimization of public building glazing according to energy efficiency criteria
Leonid Kosenko, Olena Koval, Yevhenii Yurchenko, Artem Koval, Serhii Hryshyn



ELECTRICAL ENGINEERING AND INDUSTRIAL ELECTRONICS

- 64 Energy conversion efficiency in the electromechanical system with magnetic gear of passenger electric transport rolling stock
Mykola Ostroverkhov, Liudmyla Spinul, Heorhii Veshchikov
- 75 Abstracts and References
- 83 Анотації



Volodymyr Kovbasa,
Nataliia Priliepo

ANALYTICAL DETERMINATION OF ABSOLUTE DEFORMATIONS AND BOUNDARIES OF THE CONTACT ZONE OF A DEFORMABLE WHEEL WITH A DEFORMABLE SURFACE

The object of research is the contact interaction between a deformable wheel (tire) and a deformable supporting surface. One of the most problematic areas is defining the boundaries of the contact zone as a function of applied loads, wheel geometry, and the mechanical properties of the contacting surfaces, as well as the increasing need to reduce energy consumption and soil compaction in the context of vehicle running gear systems.

During the research, methods from the mechanics of continuous deformable media were used to identify the deformable properties of contacting surfaces. The investigations were founded on previously published research concerning the distribution of contact forces, obtained using biharmonic potential analytical functions based on methods employed by Boussinesq and Cerruti.

Analytical dependencies have been derived that relate the leading and trailing edges of the deformable wheel's (tire's) contact with the deformable surface. This is because the proposed approach, which considers the relationship between forces and displacements on the surface and deformations and stresses within the contacting bodies, taking into account their mechanical properties, is more informative compared to methods of theoretical and analytical mechanics. The research revealed that the potential for improving the object's functioning lies in optimizing the selection of parameters and operating modes for running gear systems.

Thanks to this, it becomes possible to obtain optimized values for the geometric size of the contact surface. Compared to analogous known approaches, this ensures an enhancement in the tractive efficiency of driving wheels, minimizing wheel slip to acceptable levels, which directly addresses key challenges in agricultural and off-road machinery by contributing to reduced energy consumption and soil compaction.

Keywords: deformable wheel, deformable support surface, absolute deformations, contact zone boundaries.

Received: 29.09.2025

Received in revised form: 23.11.2025

Accepted: 14.12.2025

Published: 29.12.2025

© The Author(s) 2025

This is an open access article
under the Creative Commons CC BY license
<https://creativecommons.org/licenses/by/4.0/>

How to cite

Kovbasa, V., Priliepo, N. (2025). Analytical determination of absolute deformations and boundaries of the contact zone of a deformable wheel with a deformable surface. *Technology Audit and Production Reserves*, 6 (1 (86)), 26–33. <https://doi.org/10.15587/2706-5448.2025.346717>

1. Introduction

The accuracy in defining the boundaries of the contact zone between deformable bodies with individual geometries in contact mechanics is a decisive part of many thematic publications. Quite logically, within this contact zone, the interacting bodies are under deformation, so the challenge lies in the correct analysis of the existing boundaries, especially when the interacting bodies have different Poisson's ratios and moduli of elasticity.

The basic principles, mathematical tools, and theories can be found in the fundamental work of contact mechanics [1] that influenced the development of existing methods for determining the contact zones [2]. Empirical methods use experiments that are done under specific conditions of load, material properties, and geometry. Well-known numerical methods, such as FEM (Finite Element Method) or DEM (Discrete Element Method) are able to determine specific contact characteristics for the conditions mentioned above. Both methods are widely used.

For example, this study [3] showed an interesting experiment for observing a tire deformation in real time. A system of high-speed cameras and a specially made stability aid was used to create a 3D profile of the tire and its deformations, showing promising data for characteristics of the contact zone. Such experiments are important for getting results

in real-life conditions, cause later they can be used for more in-depth numerical models. In contrast, they are often focused on developing simulations used for explaining the tire-soil interactions [4]. These simulations can be proven accurate through comparison with existing test results for soil particle mobility that show significant likeness between empirical and numerical data. At the same time, formulating general dependencies for contact zone interactions needs using both extensive experiments and difficult mathematical calculations that are inconvenient and excessive [5]. Furthermore, the reliability of the obtained results can vary due to the starting hypothesis.

Analytical methods for solving elastic contact problems often use the functions of Boussinesq and Cerruti. They provide basic mathematical solutions for concentrated normal and tangential loads, showing an important beginning for later analysis [6]. All these studies are used for further advancement in the circuit. Specific geometries, material properties, and applied loads are the basis for modelling equations for contact zones and rolling resistance of the wheel-support surface interactions [7]. A considerably new study shows how using biharmonic functions that take into account equations for distributed and concentrated forces can determine the force distribution at the contact zone [8].

But using only simplified assumptions and selective omissions can significantly influence the predictive accuracy. This study [9] demonstrates

valid experimentation that shows how ESWL (Equivalent Single-Wheel Load) simplifies pavement design, combining multiple loads into one, neglecting critical present longitudinal and transversal stresses. In the same manner, existing empirical models can be found efficient, but lacking in taking into consideration complex non-linear relationships in deformations in the wheel-soil contact zone, potentially leading to faulty mobility predictions [10].

One of the most important parameters of possible solutions is the boundaries of the contact zone, because of its significance in describing force distributions, body deformation, and internal stresses. These boundaries are, in turn, functions of the applied loads, the mechanical properties, and geometry of the interacting bodies. Therefore, developing a sound method for determining these boundaries is pivotal for the circuit of contact mechanics.

Thus, the *object of research* is the contact interaction between a deformable wheel (tire) and a deformable supporting surface.

The *aim of research* is to establish an analytical relationship for determining the boundaries of the contact zone as a function of applied loads, the geometric parameters of the wheel, and the mechanical properties of the contacting surfaces.

The tasks of this research were as follows:

- 1) to determine the absolute vertical components of deformation for both the tire and the supporting surface, based on the dependencies presented in the previous publication;
- 2) to ascertain the magnitude of the maximum absolute vertical deformation of the deformed tire at the trailing edge of the contact patch, considering that the deformation of the supporting surface at this point is finite;
- 3) to determine the horizontal boundary of the finite tire deformation at the trailing edge of the contact patch;
- 4) to determine the leading edge of the contact zone, given the known magnitude of the trailing edge and considering that the cumulative deformation of the tire and the supporting surface at this point is zero and derive the final equations for the boundaries of the leading and trailing contact points.

2. Materials and Methods

This research extends the work initiated in paper [8] on the contact interaction between a deformable wheel (tire) and a deformable surface. The primary objective of this extension is to establish analytical expressions for the absolute deformations of both the wheel and the supporting surface. These expressions are critical, as they can be analytically transformed to define the boundaries of the contact zone as a function of applied loads, wheel geometry, and the mechanical properties of the interacting materials.

The methodology leverages analytical force distributions previously developed for non-conforming bodies. These are integrated with transformed Boussinesq and Cerruti equations [8] within a planar problem formulation. This synthesis yields the components of absolute deformation for both the wheel and the supporting surface. Crucially, these deformation components are biharmonic analytical functions, representing the solution to the first boundary value problem for a half-space. As the initial force distributions are functions of the system's loads, geometry, and material properties, the resulting deformation fields are likewise dependent on these same parameters.

This analytical approach offers significant advantages over purely numerical or experimental methods. It provides a more general solution, offers deeper insight into the underlying physical mechanisms of contact, and furnishes a robust foundation for subsequent theoretical developments and parametric studies.

The general case of the interaction between a deformable wheel and soil, which considers all the acting forces, is shown in Fig. 1. This model can be used for both driving and passive wheels.

The mathematical equations for absolute surface deformation previously published in paper [7], made for a planar problem, can be used as the basis for created analytical relationships for displacement. At the same time, these relationships allow for a systematic analysis of the resulting deformation. They also can look into key parameters and surface properties that influence these deformations.

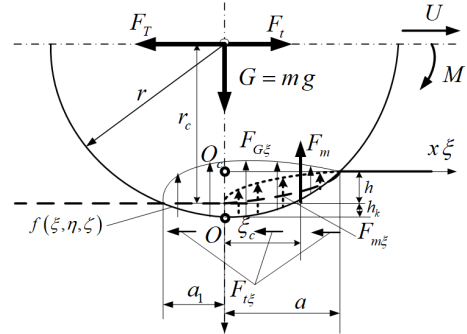


Fig. 1. Schematic of the wheel-soil interaction, illustrating the forces in the general case for both driving and passive wheel conditions

Fig. 1 illustrates the interaction scheme of a deformable wheel within the $\xi\zeta$ coordinate system, which coincides with the xz coordinate system of the deformable surface. The division of the coordinate systems is made to allow for the description of the geometric parameters of both the wheel and support surfaces.

The following forces act on the wheel's center of rotation: the weight acting on the wheel $G = mg$, the pushing force (for a passive wheel) exerted from the frame F_p , the traction force applied to the wheel from the frame caused by the tractive force F_T , and the torque that causes the wheel to rotate (for the active wheel, for the passive wheel $M = 0$).

The distributed forces in the wheel-surface contact zone are designated as follows: F_m – wheel rolling resistance, where the action center is located at a ξ_c distance from the wheel axis. The symbol U indicates the direction of the wheel's movement.

The geometric parameters of the contact zone interaction are as follows: a, a_1 – the front and the rear contact boundaries, h – the support surface deformation, h_k – deformation of the deformable wheel [8]. The presented solution is a continuation of the previously published study [8].

3. Results and Discussion

Vertical displacements of the wheel surface:

$$w_{k\xi\eta} = -\frac{1}{4G_k\pi} \times \int_{b_1}^b \int_{a_1}^a \left(1 - \frac{2\xi^2}{r^2} - \frac{2k^4\eta^2(r^2 - 6\xi^2)}{r^4} \right) \left(\frac{F_{zkk}(-3 + 2\nu_k)}{\sqrt{z^2 + (y - \eta)^2 + (x - \xi)^2}} + \frac{(-1 + 2\nu_k)F_{xkk}}{\sqrt{z^2 + (y - \eta)^2 + (x - \xi)^2}} \right) d\xi d\eta; \quad (1)$$

$$F_{xkk} = \frac{F_t - F_T + ftgm + \frac{M_c}{h_k - r}}{(a - a_1)(b - b_1)},$$

$$F_{zkk} = \frac{-a_1M_c + a^2 \left(F_{T_k} + \frac{a_1F_t}{h_k - r} \right) + a \left(-a_1(F_{T_k} + gm) + M_c + \frac{a_1^2F_t}{-h_k + r} \right)}{a(a - a_1)a_1(b - b_1)},$$

where G_k – elastic modulus of the wheel material; a, a_1 – front and rear contact boundaries; ν_k – Poisson's ratio of the wheel material; b, b_1 – transverse wheel boundaries (axis Oy idem $O\eta$); r – wheel radius;

ξ – longitudinal coordinate of the deformable wheel system; k – coefficient characterizing the ellipticity of the contact zone surface; η – transverse coordinate of the deformable wheel system; x – longitudinal axis of the coordinate system under consideration; y – transverse axis of the coordinate system under consideration; z – vertical axis of the coordinate system under consideration; d – final total deformation of the supporting surface; F_{zkk} – component of the total function for the vertically distributed forces acting on the wheels; F_{xkk} – component of the total function for the horizontally distributed forces acting on the wheels; F_T – tractive force; F_t – pushing force; g – gravitational acceleration; m – mass per wheel; M_c – torque; h_k – component of the final deformation of the deformable wheel.

The fully expanded analytical solution to this integral is omitted due to its considerable length and complexity. However, the solution can be evaluated numerically to visualize the deformation profile. Fig. 2 illustrates the resulting vertical displacement of the wheel surface across the contact zone.

An analysis of the vertical wheel deformation, governed by (1) and illustrated in Fig. 2, reveals the influence of key system parameters on the magnitude of displacement. The primary relationships observed are that the magnitude of the wheel's absolute surface deformation increases with increases in the applied torque M_c , tractive force F_T , and mass m . Conversely, an increase in the wheel's surface elastic modulus G_k results in a decrease in the vertical deformation component.

Vertical displacements of the support surface in the contact zone:

$$w_{n\xi\eta} = \int_{b_1}^b \int_{a_1}^a \left(\frac{2\eta^2 k^4 (r^2 - 6\xi^2)}{r^4} - \frac{F_{xnm} (2\nu_n - 1)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}} + \frac{F_{znm} (2\nu_n - 3)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}} \right) d\xi d\eta$$

$$= - \frac{1}{4\pi G_n} \left(\frac{F_t - F_T + \frac{M_c + f_i g m (h-r)}{h-r}}{(a-a_1)(b-b_1)} - \frac{a_1^2 (F_t + F_T) + 2a_1 M_c (-h+r) - a_1 g m (h-r) + 2M_c (-h+r)}{a(a-a_1)a_1(b-b_1)(-h+r)} \right) \quad (2)$$

where G_n – elastic modulus of shear deformation of the support surface; ν_n – Poisson's ratio of the support surface; F_{xnm} – component of the total function for the horizontally distributed forces acting on the supporting surface; F_{znm} – component of the total function for the vertically distributed forces acting on the supporting surface, h – component of the final deformation of the deformable supporting surface.

Fig. 3 illustrates the resulting vertical displacement of the support surface across the contact zone.

Analysis of the vertical deformation of the support surface, as described by (2) and visualized in Fig. 3, yields the following key findings: the absolute deformation of the support surface increases with an increase in any of the applied loads. This includes the wheel torque M_c , tractive force F_T , pushing force F_t (for a passive wheel), and mass m per wheel. An increase in the elastic modulus of the support surface G_n results in reduced vertical deformation. As expected, a stiffer surface material is more resistant to indentation under load.

Increasing the wheel radius r leads to a decrease in deformation of the support surface. This occurs because a larger radius distributes the vertical load over a longer contact patch, which reduces the peak contact pressure exerted on the surface.

Horizontal movements of the wheel surface:

$$u_{k\xi\eta} = - \frac{1}{4G_k \pi} \times \int_{b_1}^b \int_{a_1}^a \left(\frac{1 - \frac{2\xi^2}{r^2}}{2k^4 \eta^2 (r^2 - 6\xi^2)} - \frac{F_{xkk} (2\nu_k - 1)}{\sqrt{z^2 + (y-\eta)^2 + (x-\xi)^2}} + \frac{F_{zkk} (2\nu_k - 3)}{\sqrt{z^2 + (y-\eta)^2 + (x-\xi)^2}} \right) d\xi d\eta$$

$$F_{xkk} = \frac{F_t - F_T + f_i g m + \frac{M_c}{h_k - r}}{(a-a_1)(b-b_1)},$$

$$F_{zkk} = \frac{-a_1 M_c + a^2 \left(F_T + \frac{a_1 F_t}{h_k - r} \right) + a \left(\frac{-a_1 (F_T + g m)}{+ M_c + \frac{a_1 F_t}{-h_k + r}} \right)}{a(a-a_1)a_1(b-b_1)} \quad (3)$$

Fig. 4 illustrates the horizontal displacement profile of the wheel surface within the contact zone.

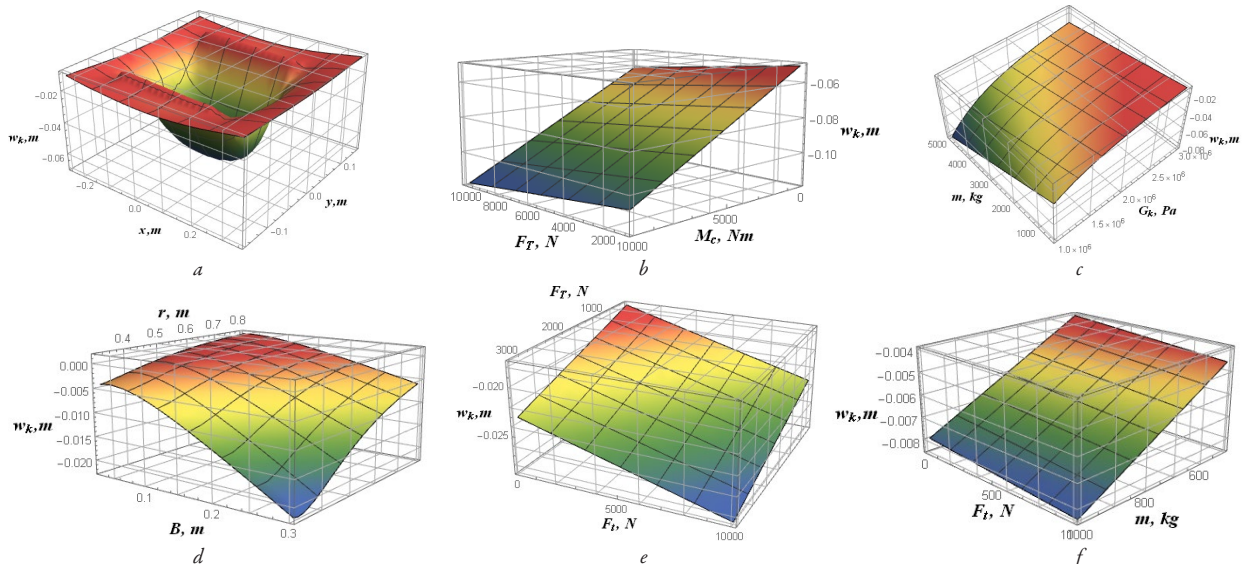


Fig. 2. Graphical representation of the vertical displacement profile of the wheel surface within the contact zone by its parameters and properties: *a* – 3D visualization of the vertical displacement of the wheel surface; *b* – influence of applied torque and traction force; *c* – influence of mass and elastic modulus of the wheel; *d* – influence of width and radius of the wheel; *e* – influence of traction and pushing forces; *f* – influence of mass and pushing force

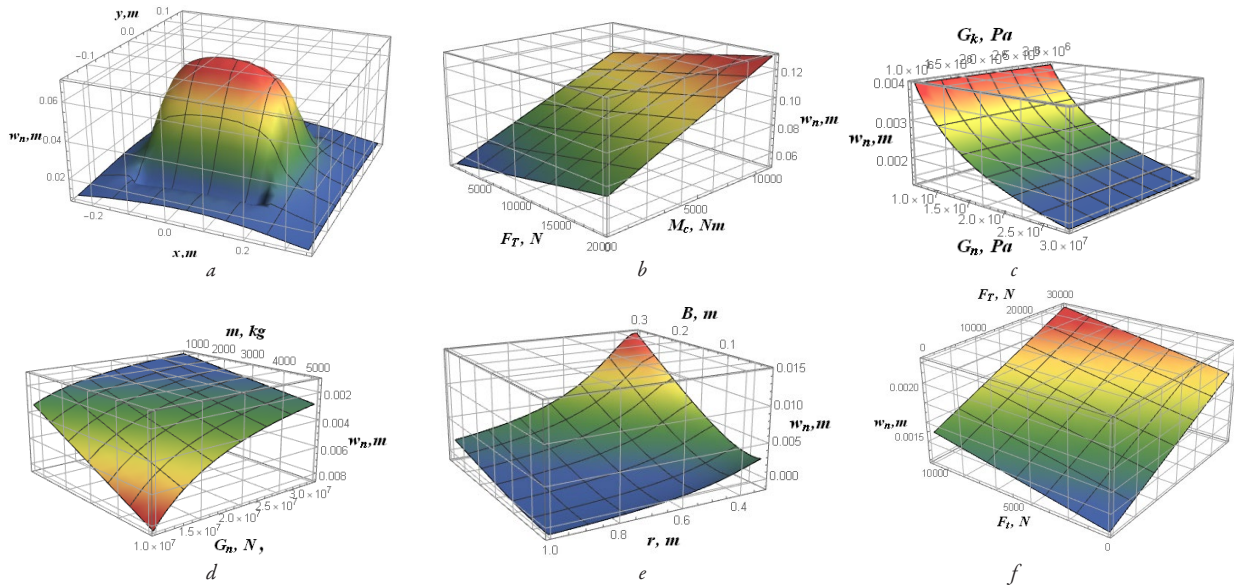


Fig. 3. Graphical representation of the vertical displacement profile of the support surface within the contact zone by its parameters and properties: *a* – 3D visualization of the vertical displacement profile of the support surface; *b* – influence of applied torque and traction force; *c* – influence of elastic moduli of the wheel material and support surface; *d* – influence of mass and elastic modulus of the support surface; *e* – influence of width and radius of the wheel; *f* – influence of traction and pushing forces

Arriving from (3), Fig. 4 demonstrates the following dependencies: horizontal deformations increase with the increase in the applied torque M_c , tractive force F_T , pushing force F_p (for a passive wheel), and mass m per wheel. This increase will cause the wheel to deform horizontally, as it'll initiate the shear stresses in the contact zone.

Contrarily, an increase in the wheel's elastic modulus G_k or its radius r will lead to a decrease in horizontal deformations, as a larger radius can change the distribution of shear stresses.

Horizontal displacements of the support surface in the contact zone:

$$u_{n\zeta\eta} = \int_{b_1}^1 \int_{a_1}^a \left(\frac{2\eta^2 k^4 (r^2 - 6\xi^2)}{r^4} - \frac{2\xi^2}{r^2} + 1 \right) \left(\frac{F_{xm}(2\nu_n - 3)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}} + \frac{F_{xm}(2\nu_n - 1)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}} \right) d\xi d\eta$$

$$= - \frac{F_{xm}}{4\pi G_n} \left(\frac{F_T - F_p + \frac{M_c + f_p g m (h-r)}{h-r}}{(a-a_1)(b-b_1)} \right)$$

$$F_{xm} = \frac{a^2 a_1 (F_T + F_p) + 2a_1 M_c (-h+r) - a \left(a_1^2 (F_T + F_p) + a_1 g m (h-r) + 2M_c (-h+r) \right)}{a(a-a_1)a_1(b-b_1)(-h+r)} \quad (4)$$

Fig. 5 illustrates the horizontal displacement profile of the support surface within the contact zone.

Analyzing Fig. 5, arriving from (4), it's possible to grasp that an increase in the wheel torque M_c , the resultant tractive force F_T , the pushing force in the case of a passive wheel F_p , or the mass m per wheel leads to a more noticeable horizontal deformation of the support surface. This can be seen as a direct result of the increase in shear stress acting in the contact zone, which is needed to create traction and overcome the rolling resistance. The illustrations also show that some parameters can have an abating effect. For example, an increase in the elastic modulus of the support surface G_n boosts the material's natural resistance to

shear, reducing the horizontal displacement. Similarly, increasing the wheel radius r will distribute the contact pressures and shear stresses over a larger area, which will lead to a decrease in the top deformation of the support surface.

The longitudinal range on the contact zone of the deformable wheel with the deforming surface is along the axis $Oy_{idemOn\eta}$. This allows to look at the existing problem as a planar problem formulation. It will also simplify the equations, because under the conditions of the problem, on the rear (trailing) edge of the contact zone a_1 , that is, when $x = \xi = a_1$, the maximum displacement of the wheel surface in the first approximation is $w_k|_{\xi=x=a_1} = a_1^2/2r$.

At the leading edge of the contact zone $x = \xi = a$, the sum of the surface displacements represents the total gap closure and is equal to zero: $w_k + w_n|_{\xi=x=a} = 0$.

Determining the contact zone boundaries requires evaluating the vertical displacement fields for both the wheel and the bearing surface at the specified points.

To calculate the boundaries of the contact zone between the wheel and the support surface, it is necessary to determine the components of the vertical displacements of the wheel surface and the support surface at the contact zone boundary points $x = \xi = a_1$ and $x = \xi = a$. The vertical component of the wheel displacement is simplified under the assumption of small deformations. This allows the higher-order terms in the expression to be neglected, resulting in the form presented in (1), and given that $E = 2(1 + \nu)G$, the last expression will take the form:

$$w_k = \frac{(1 + \nu_k) \xi \left(3F_{xkk} \pi (-1 + 2\nu_k) \xi (-r^2 + \xi^2) + F_{zkk} (-1 + \nu_k) \left(12x^3 + 6x^2 \xi + 4x \xi^2 + 3\xi^3 - 6r^2 (2x + \xi) \right) \right)}{12E_k \pi r^2}$$

$$w_k a_1 = (w_k / \xi = a_1) - (w_k / \xi = a_1) \Big|_{x=\xi} =$$

$$a_1 (1 + \nu_k) \left(\frac{F_{zkk} (3a_1^3 - 6a_1 r^2) (-1 + \nu_k) + 3a_1 F_{xkk} \pi (a_1^2 - r^2) (-1 + 2\nu_k)}{12E_k \pi r^2} \right) \quad (5)$$

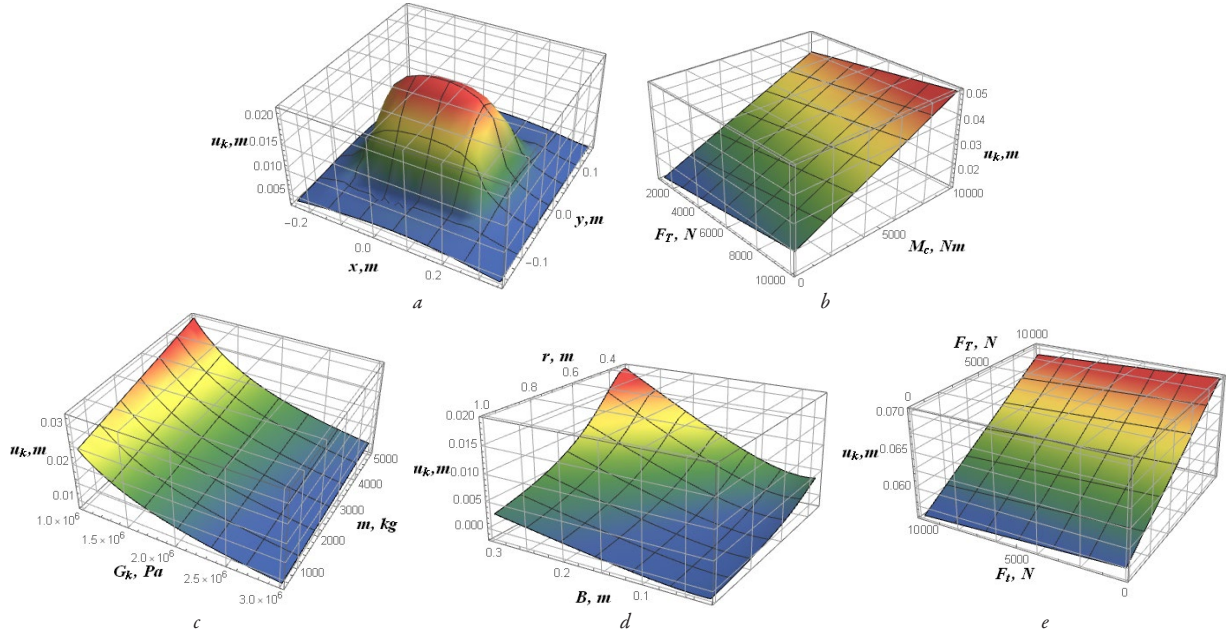


Fig. 4. Graphical representation of the horizontal displacement profile of the wheel surface within the contact zone by its parameters and properties: *a* – 3D visualization of the horizontal displacement of the wheel surface; *b* – influence of applied torque and traction force; *c* – influence of mass and elastic modulus of the wheel; *d* – influence of width and radius of the wheel; *e* – influence of traction and pushing forces

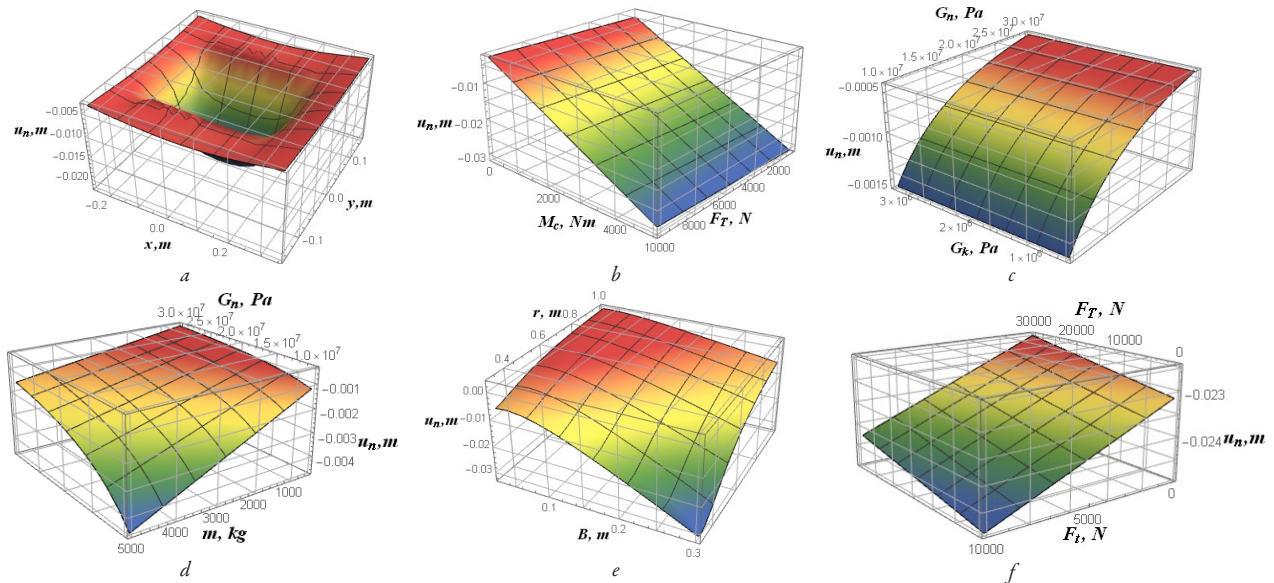


Fig. 5. Graphical representation of the horizontal displacement profile of the support surface within the contact zone by its parameters and properties: *a* – 3D visualization of the horizontal displacement profile of the support surface; *b* – influence of applied torque and traction force; *c* – influence of elastic moduli of the wheel material and support surface; *d* – influence of mass and elastic modulus of the support surface; *e* – influence of width and radius of the wheel; *f* – influence of traction and pushing forces

Equating the previously derived expression with the vertical wheel deformation at the point a_1 : $w_{k|z=x=a_1} = a_1^2/2r$, and subsequently solving the resulting equation for a_1 , it is possible to obtain an expression for the coordinate of the rear boundary of the contact zone

$$a_1 = \left(\frac{2a(\pi BE_k(h_k - r) - F_t(v_k^2 - 1)r) - (v_k + 1)r \left(gm(\pi f_t(2v_k - 1) - 2v_k + 2) \right)}{\times (aF_t + M_c)} \right) \times \left(\frac{F_t(h_k - r) + (F_t(r - h_k) + M_c)}{\times (h_k - r) + \pi(2v_k - 1)} \right) \quad (6)$$

The dependency of the rear contact boundary's location on key wheel parameters is illustrated in Fig. 6.

The analysis of the dependence illustrates that:

- with an increase in the wheel radius r , the length of the rear boundary of the contact zone a_1 increases linearly;
- with an increase in the wheel's modulus of curvature E_k , the contact area a_1 decreases inversely;
- with increasing tractor mass applied to the wheel m , the size of the rear boundary of the contact area a_1 decreases linearly;
- with increasing tractive force F_t , the coordinate of the rear boundary of the contact area a_1 increases linearly;
- with a decrease in the width of the wheel B , the coordinate of the rear boundary of the contact zone a_1 increases inversely;
- an increase in the applied torque M_c leads to a non-linear increase in the rear boundary coordinate a_1 .

To determine the front boundary of the contact zone, the functions of the sum of the vertical displacements of the wheel and the

bearing surface from (1) and (2) must be reduced to zero at $x = \zeta = a$. An expression for the coordinate of the front boundary is derived by applying the governing contact condition $w_k + w_n|_{\zeta=x=a} = 0$ and using the solution for the rear boundary a_1 . This final expression is simplified by neglecting higher-order terms, which are considered insignificant for this analysis, but taking into account that $E = 2(1 + \nu)G$

$$a = - \left(\frac{2M_c(h-r)(r-h_k) \times \left(\frac{2E_k(vn^2-1) \times}{+E_n(vk^2-1)} \right)}{-2a_1 \times (F_i + F_T)(h_k-r) + E_n F_i (vk^2-1)(h-r)} \right) + E_k(vn+1)(h-r) \left(\frac{-gm(\pi f_i(2vn-1) + 2(vn-1))(h-r) - \pi(2vn-1)(F_i(h-r) - F_i h + F_i r + M_c)}{-\pi(2vk-1)(F_i(h_k-r) + M_c) - F_T(2\pi vk + 2vk - \pi - 2)(h_k-r)} \right) + E_n(vk+1)(h-r) \left(\frac{gm(\pi f_i(2vk-1) - 2vk + 2)(h_k-r) + \pi(2vk-1)(F_i(h_k-r) + M_c) - F_T(2\pi vk + 2vk - \pi - 2)(h_k-r)}{\dots} \right). \quad (7)$$

Fig. 7 illustrates how the location of the front boundary varies as a function of the wheel's geometric and material properties.

The analysis of the dependence illustrates that:

- the modulus of wheel's curvature, E_m , is inversely proportional to the front boundary coordinate of the contact zone a ;
- the front boundary of the contact zone, a , increases linearly with the applied torque M_c ;
- the front boundary of the contact zone, a , increases linearly with the wheel's modulus of curvature E_k ;
- the front boundary of the contact zone, a , is inversely proportional to the wheel radius r ;
- the front contact boundary, a , is inversely proportional to the mass of the tractor, m , applied to the wheel;
- the position of the front contact boundary, a , shows a slight positive dependence on the applied pushing force F_i ;
- the front contact boundary, a , decreases linearly as the tractive force F_T , increases.

As a result of the conducted research, analytical relationships have been derived to determine the boundaries of the contact zone as

a function of applied loads, geometric parameters of the wheel, and the mechanical properties of the contacting surfaces. Based on the solutions obtained for this research objectives, the following conclusions can be drawn. The obtained analytical relationships significantly simplify the prediction of the nature of the contact interaction between two deformable bodies of non-conformal geometry. This offers a significant advantage compared to known methods of experimental research and numerical modelling using FEM and FVM methods. The latter requires substantial material costs (for experimental methods) and considerable computations (for numerical methods). The results obtained enable the investigation of slippage and sliding conditions of wheeled propulsion systems and the determination of their trafficability, given the known mechanical properties of the support surface.

The research findings presented in this publication can be practically applied to determine the dimensions of the contact patch between a deformable wheel (tire) and a deformable supporting surface, given known wheel loads, geometric dimensions, and the mechanical properties of both the tire and the supporting surface. This enables the determination of permissible loads on the supporting surface, the wheel's tractive characteristics, and allowable deformations to ensure the requisite off-road capability and tractive effort that the unit can generate.

Furthermore, in comparison to numerical solution methods, they – while possessing lower costs associated with the absence of extensive preparatory work and calculations – have sufficient accuracy. They also offer the possibility of analyzing the influence of various factors on the result and obtaining the expected nature of contact interaction at the design stage or for ensuring rational machine operating modes.

The results of the presented research open the possibility for theoretical investigations of slippage and sliding issues in the wheel-support surface contact zone, as well as for determining machine trafficability and the rational use of mobile machinery.

The research results possess certain limitations, primarily related to the necessity of determining the mechanical properties of the tire and the supporting surface. Furthermore, applying these results requires a specialist to possess knowledge of continuum mechanics for characterizing the mechanical properties of the contacting bodies, along with a university-level understanding of mathematics relevant to technical fields.

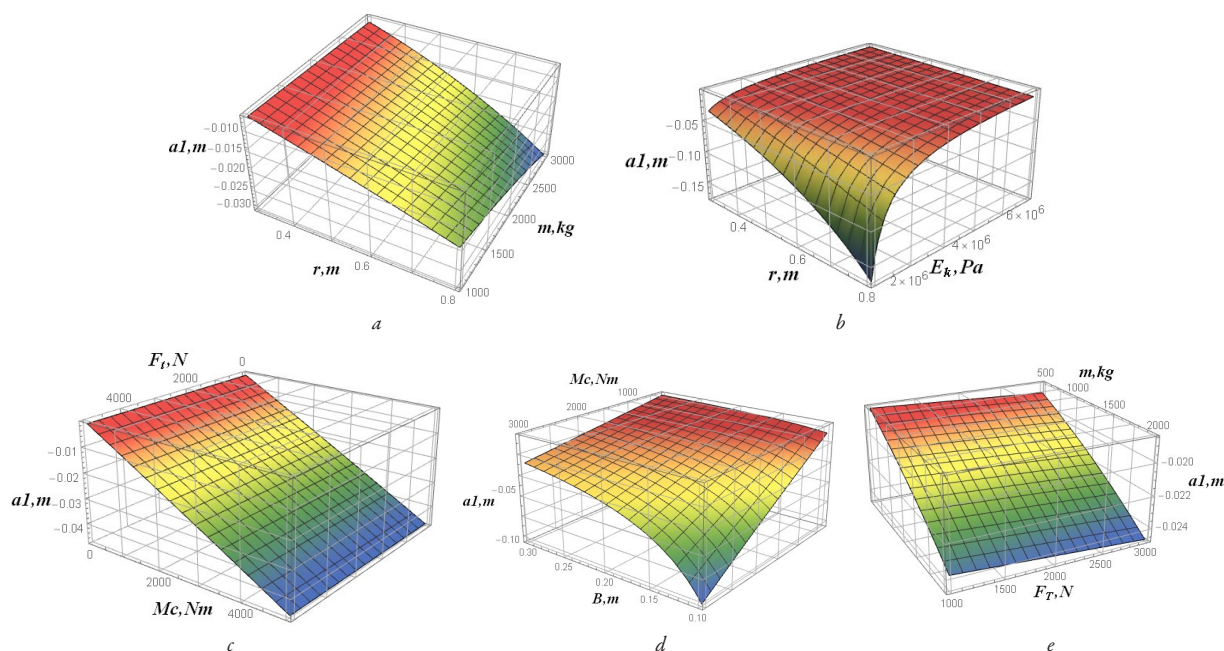


Fig. 6. Graphical representation of dependence of the rear contact boundary coordinate a_1 on key wheel parameters and properties: a – influence of wheel radius and mass; b – influence of wheel radius and wheel's modulus of curvature; c – influence of pushing force and applied torque; d – influence of applied torque and the width of the wheel; e – influence of traction force and mass

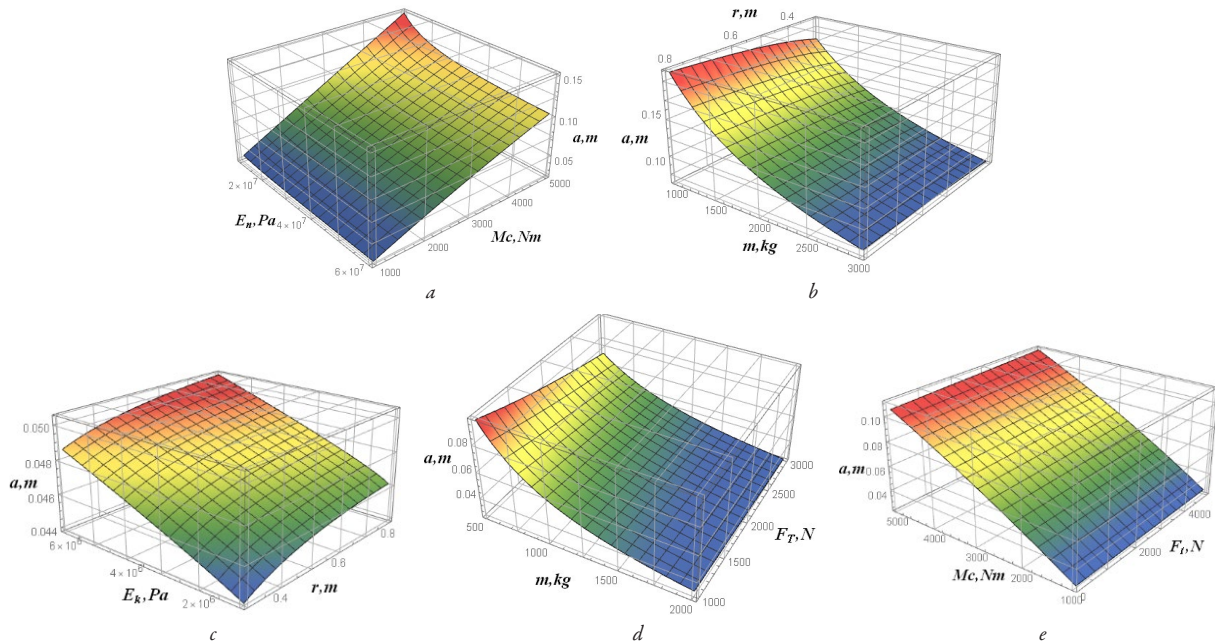


Fig. 7. Graphical representation of dependence of the front contact boundary coordinate a on key wheel parameters and properties:
 a – influence of wheel's curvature and applied torque; b – influence of wheel radius and mass; c – influence of wheel's modulus of curvature and radius;
 d – influence of traction force and mass; e – influence of applied torque and pushing force

The continued application of these research outcomes facilitates the optimization of tractive performance in mobile power units and allows for the prediction of permissible slip characteristics for wheeled machinery. Moreover, further utilization of the obtained results will contribute to reducing soil degradation caused by tractive power units, achieved by minimizing both horizontal and vertical shear deformations in the soil.

4. Conclusions

1. The absolute vertical deformation components for the tire and the support surface, which represent biharmonic analytical functions ($F_{zkk}, F_{xkk}, h_k, F_{xnm}, F_{znm}, h$), are the solution to the first boundary value problem for the plane formulation. Since the initial force distributions are functions of system loads, geometry, and material properties, the resulting deformation fields also depend on these same parameters.

2. The absolute vertical deformation of the wheel surface increases with an increase in applied torque M_o , tractive force F_T , and the system mass m distributed to the wheel. The absolute deformation of the support surface increases with an increase in any of the applied loads – including wheel torque M_o , tractive force F_T , pushing force F_i (for a passive wheel), and the mass m acting on the wheel. As expected, surface material with a higher modulus of elasticity is more resistant to indentation under load.

3. Horizontal deformations of the wheel surface increase with an increase in applied torque M_o , tractive force F_T , pushing force F_i (for a passive wheel), and the mass m acting on the wheel. This increase leads to horizontal deformation of the wheel, as it causes shear stresses in the contact zone. An increase in wheel torque M_o , tractive force F_T , pushing force F_i in the case of a passive wheel, and the mass m acting on the wheel also leads to more pronounced horizontal deformation of the support surface.

4. The final equations for determining the front and rear boundaries of contact are functions of the loads applied to the wheel, its geometric parameters, and the mechanical properties of both the wheel tire and the support surface. Research results obtained using continuum mechanics methods and solutions derived from math-

ematical physics methods enabled the determination of analytical relationships for the absolute deformation components of both the wheel surface and the support surface. These relationships allow for the determination of contact conditions to define the nature of the contact interaction between wheeled propulsion systems and the support surface. Compared to known solution methods based on theoretical mechanics, these methods account for the influence of deformations on the nature of the interaction, yielding more accurate and adequate results. Compared to experimental methods, the presented results are less labor-intensive and possess greater solution generality.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current research, including financial, personal, authorship, or any other, that could affect the research, as well as the results reported in this paper.

Financing

The research was conducted without financial support.

Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

Grammarly (v.1.2.212.1789) – proofreading, editing. All proposed changes were checked by the authors and used based on their relative appropriateness.

Authors' contributions

Volodymyr Kovbasa: Conceptualization, Methodology, Writing – review and editing; **Nataliia Prilepo:** Formal analysis, Visualization, Writing – original draft.

References

1. Johnson, K. L. (1985). *Contact Mechanics*. London: Cambridge University Press. <https://doi.org/10.1017/cbo9781139171731>
2. Kovbasa, V. P. (2006). *Mekhaniko-tehnologichne obgruntuvannya optymizatsii robochykh orhaniv z gruntom*. [Doctoral dissertation; Natsionalnyi universytet bioresursiv i pryrodokorystuvannya Ukrainy]. Available at: <https://uacademic.info/ua/document/0506U000111#>!
3. Guthrie, A. G., Botha, T. R., Els, P. S. (2017). 3D contact patch measurement inside rolling tyres. *Journal of Terramechanics*, 69, 13–21. <https://doi.org/10.1016/j.jterra.2016.09.004>
4. Yamashita, H., Jayakumar, P., Alsaleh, M., Sugiyama, H. (2017). Physics-Based Deformable Tire–Soil Interaction Model for Off-Road Mobility Simulation and Experimental Validation. *Journal of Computational and Nonlinear Dynamics*, 13 (2). <https://doi.org/10.1115/1.4037994>
5. Kushnarov, A. S., Kochev, V. I. (1989). *Mekhaniko-tehnologicheskie osnovy obrabotki pochvy*. Kyiv: Urozhai, 144.
6. Johnson, W., Mellor, P. B. (1973). *Engineering plasticity*. London, New York: Van Nostrand Reinhold Co. Available at: <https://www.scribd.com/document/489339320/Engineering-Plasticity-by-W-Johnson-and-P-B-Mellor-pdf>
7. Kovbasa, V. P., Solomka, A. V., Spirin, A. V., Kucheruk, V. Yu., Karabekova, D. Zh., Khassenov, A. K. (2020). Theoretical determination of the distribution of forces and the size of the boundaries of the contact in the interaction of the deformable drive wheel with the soil. *Bulletin of the Karaganda University "Physics Series"*, 99 (3), 62–72. <https://doi.org/10.31489/2020ph3/62-72>
8. Kovbasa, V., Priliepo, N. (2025). Determination of analytical dependencies of distributed forces in a deformable wheel – deformable surface contact zone. *Technology Audit and Production Reserves*, 3 (1 (83)), 6–12. <https://doi.org/10.15587/2706-5448.2025.329471>
9. Yang, W., Tiecheng, S., Yongjie, L., Chundi, S. (2016). Prediction for Tire – Pavement Contact Stress under Steady – State Conditions based on 3D Finite Element Method. *Journal of Engineering Science and Technology Review*, 9 (4), 17–25. <https://doi.org/10.25103/jestr.094.04>
10. Swamy, V. S., Pandit, R., Yerro, A., Sandu, C., Rizzo, D. M., Sebeck, K., Gorsich, D. (2023). Review of modeling and validation techniques for tire-deformable soil interactions. *Journal of Terramechanics*, 109, 73–92. <https://doi.org/10.1016/j.jterra.2023.05.007>

Volodymyr Kovbasa, Doctor of Technical Sciences, Professor, Department of Mechanical and Electrical Engineering, Poltava State Agrarian University, Poltava, Ukraine, ORCID: <https://orcid.org/0000-0003-4574-5826>

Natalia Priliepo, Department of Mechanical and Electrical Engineering, Poltava State Agrarian University, Poltava, Ukraine, e-mail: natalia.pryliepo@pdau.edu.ua, ORCID: <https://orcid.org/0000-0002-4182-7405>

✉ Corresponding author