THE SIMULTANEOUS USE OF EXCEL AND GEOGEBRA TO TRAINING THE BASICS OF MATHEMATICAL MODELING

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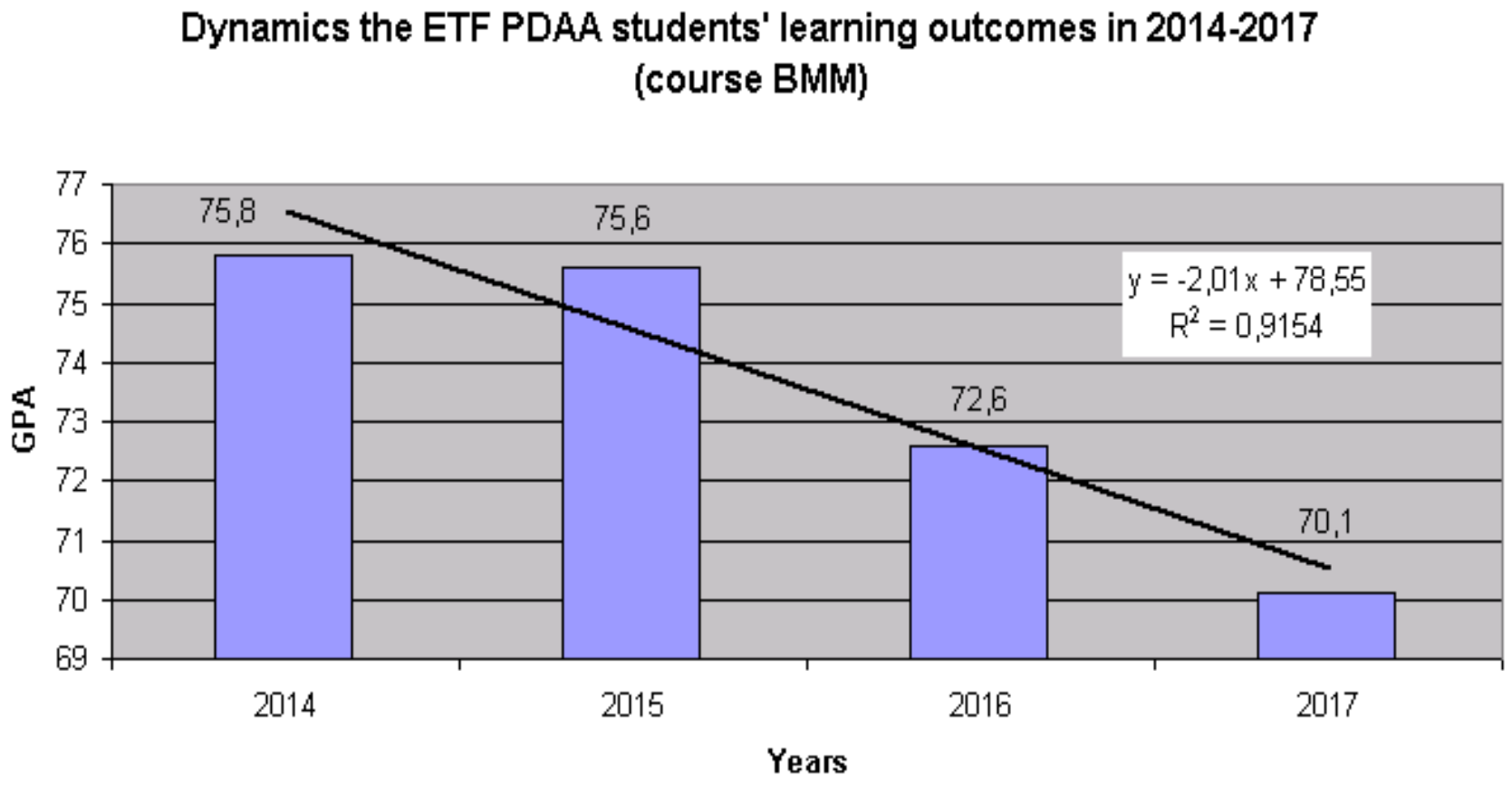
KHERSON STATE UNIVERSITY, **12.06.2019** -**15.06.2019**

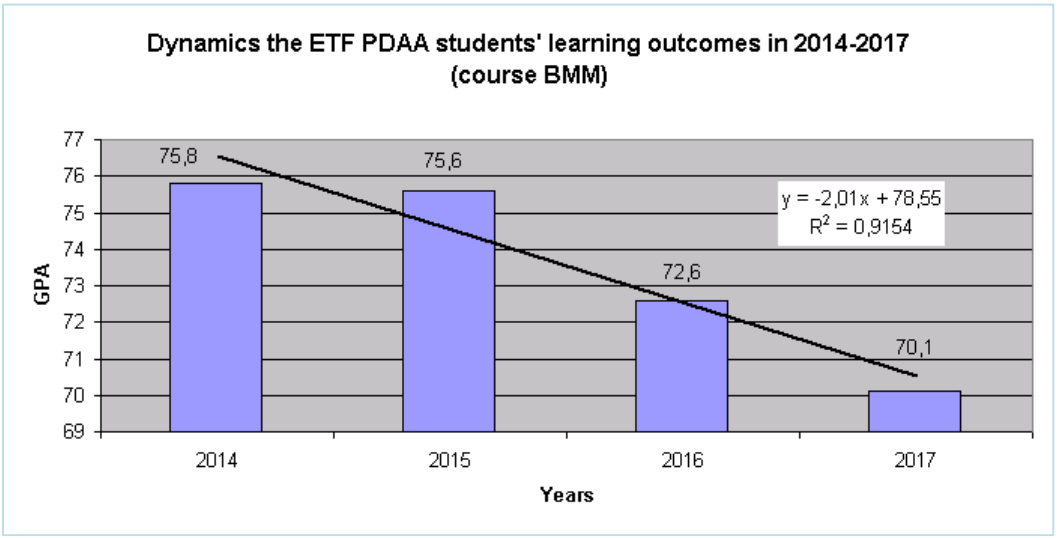

Poltava State Agrarian Academy

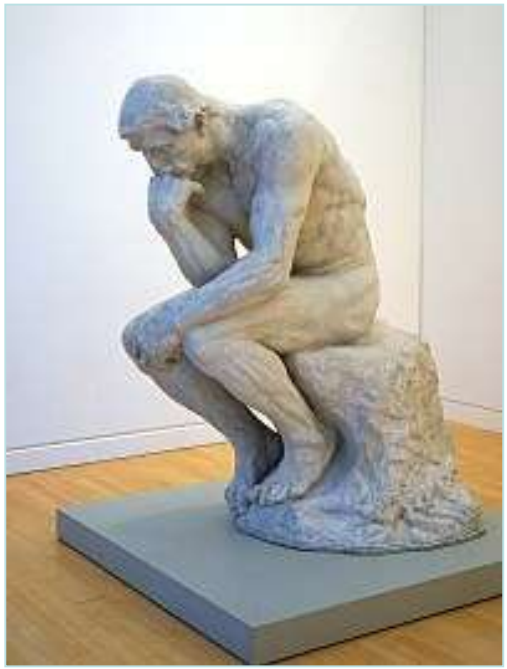
An example of real agroengineering

ETF PDAA

What was the reason for our research?







Why?

Image source: https://en.wikipedia.org/wiki/The_Thinker

What factor we had been researching?

Individual style of coding educational information

*For students of applied training areas visual learning style preferred by **61%** of the persons…*

(Aulakh, M., Khan, Z. M., Sana, A., 2018).

Image source: https://www.clipartmax.com

Hypothesis

the BMM learning outcomes can be improved by the use of appropriate computer (software) visualization tools in practical and laboratory learning performances

Objective

testing the thesis that the visualization of computer simulation results creates the conditions for improving students' knowledge

Software we used

GeoGebra

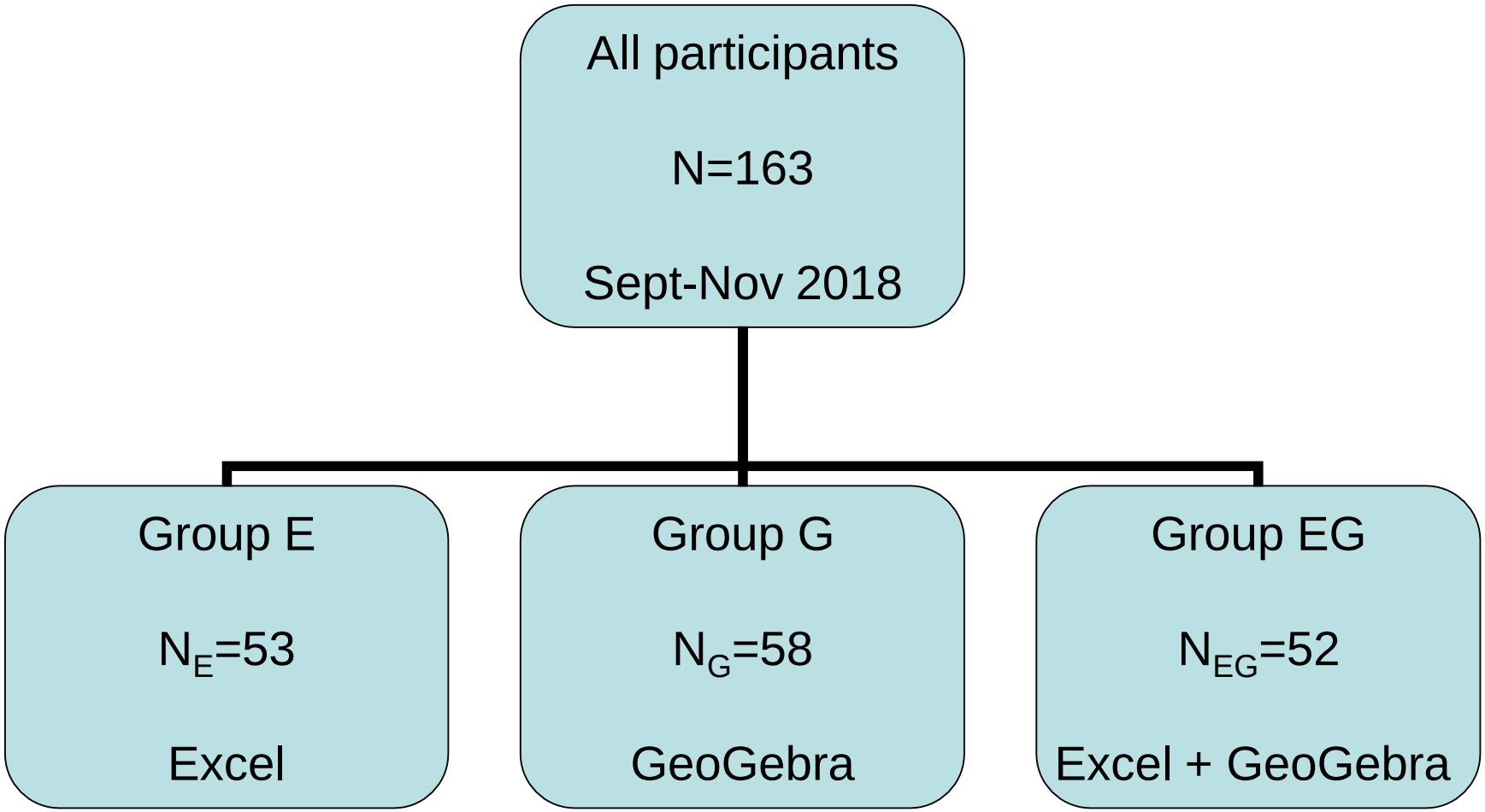
+

numerical
calculations
and graphs

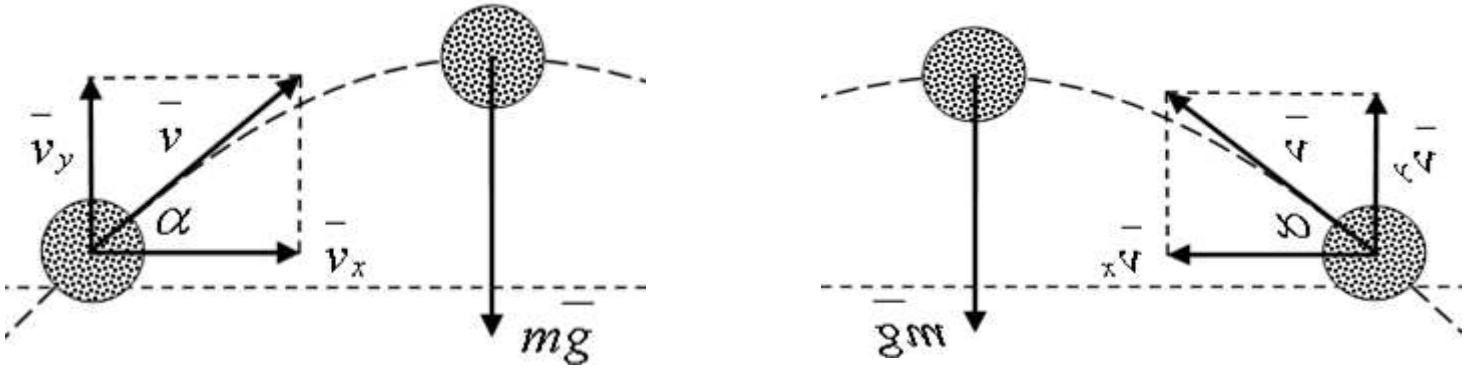
dots, lines,
vectors etc.
visualization

Images sources: http://cvolino.be; https://www.geogebra.org/geometry

Experiment design



The problem task for students



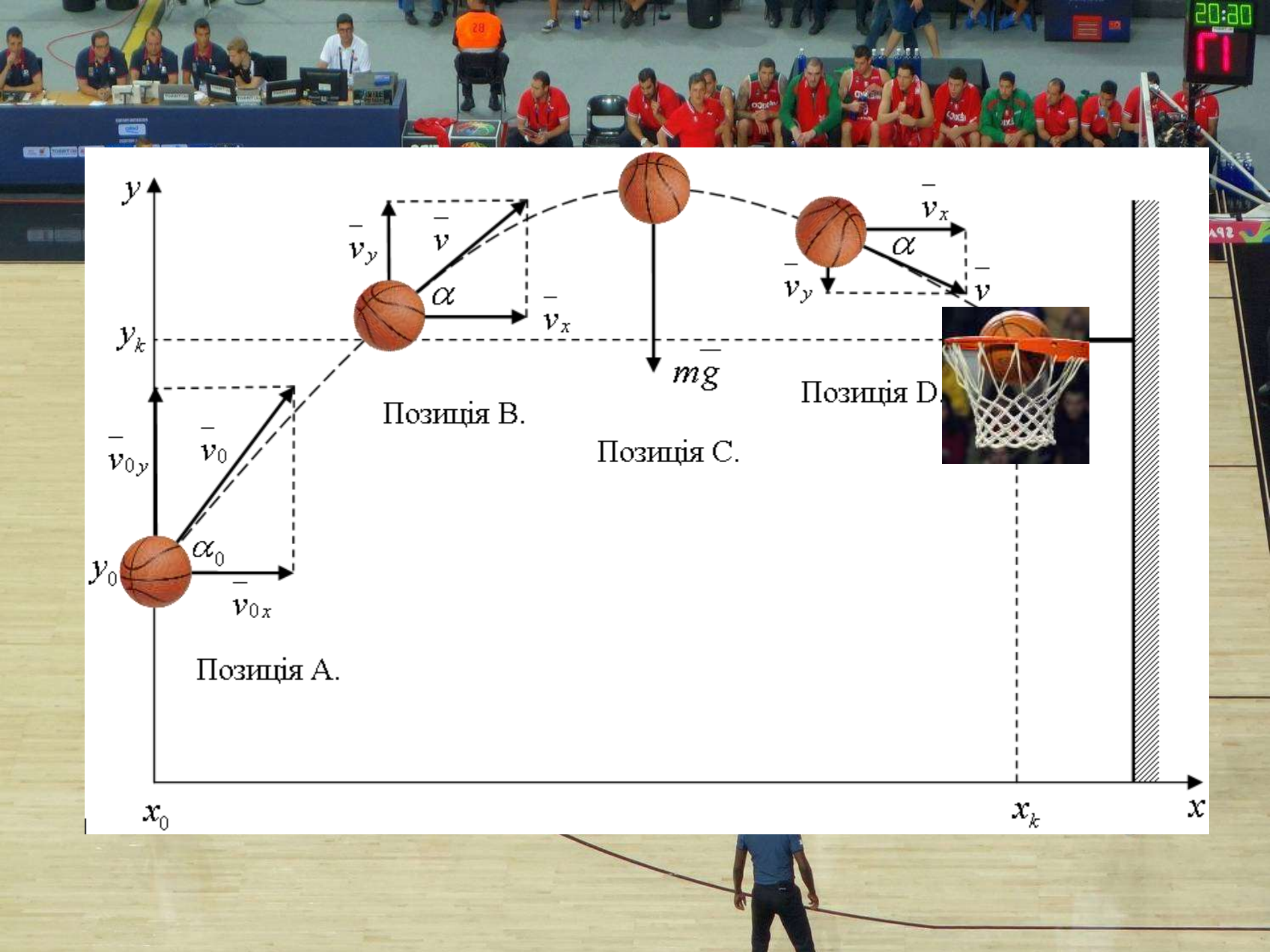
Simulate the movement of two spherical bodies were thrown at an angle to the horizon towards each other.

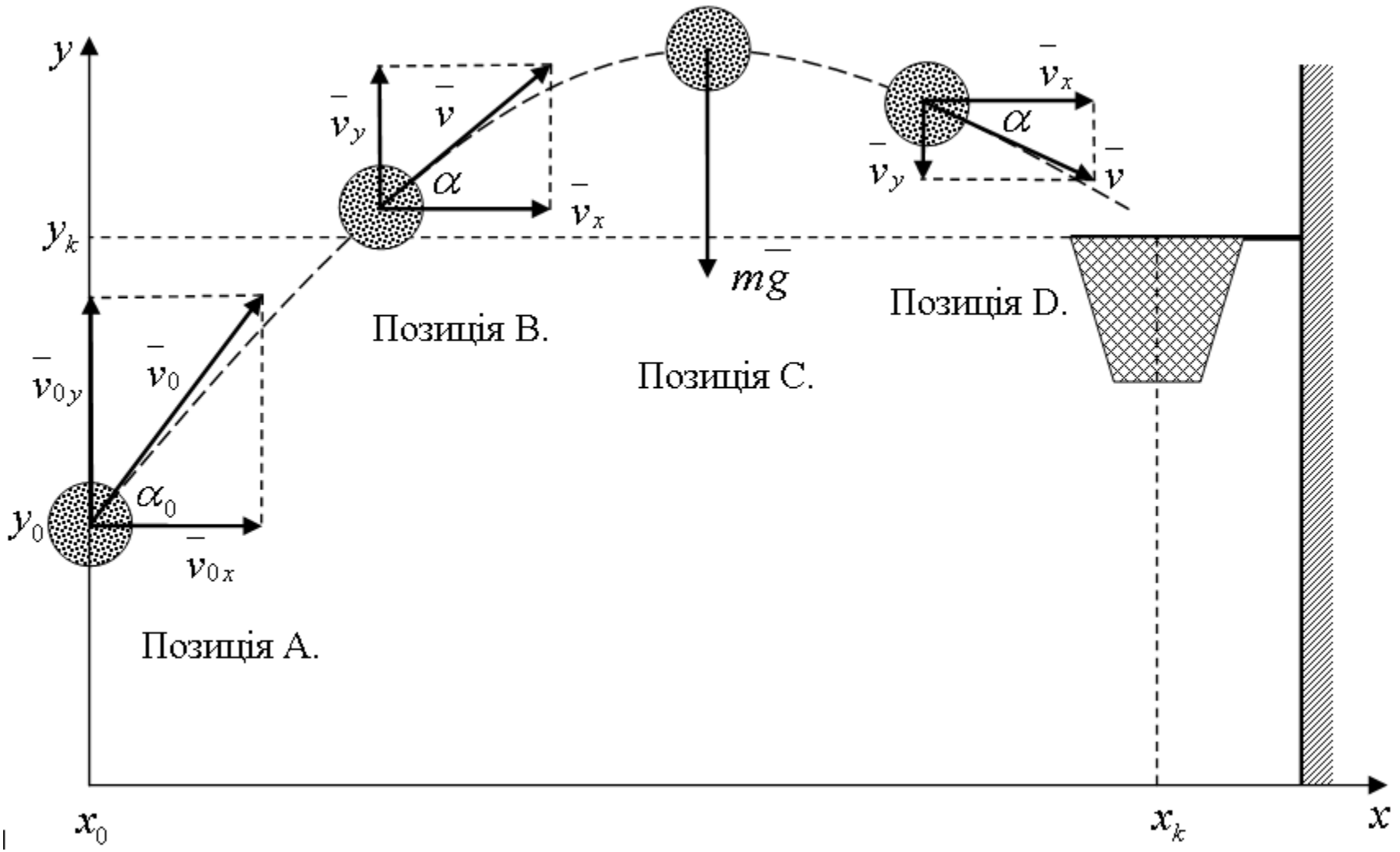
Investigate the conditions of bodies' collision if body collision is absolutely elastic.

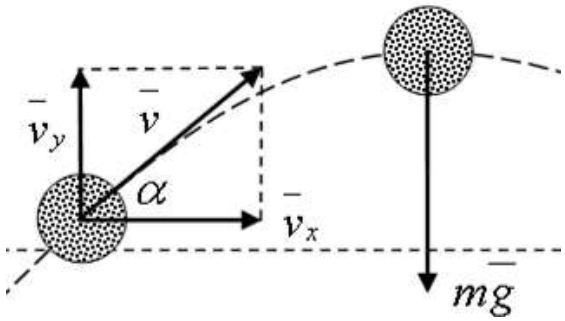
Not taking into account air resistance, gravitational and electrostatic interaction of bodies, etc.

Algorithm for solving the problem

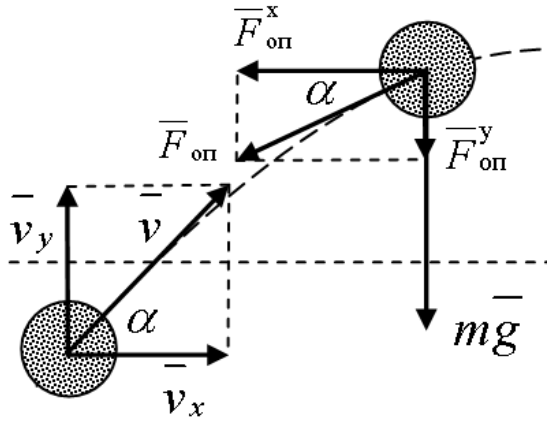
1. Build a mathematical model and find its solution.
2. Create a computer implementation of a mathematical model.
3. Using a mathematical model:
    1. Calculate the coordinates and the speeds of the bodies at given moments of time (before the collision of bodies).
    2. Build trajectories of body movement before their collision.
    3. Find the initial conditions of motion, in which there is a collision of bodies.
    4. Find: the moment of bodies' contact, coordinates of centers and speeds of bodies at the moment of their collision.
    5. Find the initial velocity of bodies after moment of their collision.
    6. Find trajectories of body movement after their collision.


y
y_k
y_0
$\bar{v}_{0y}$
$\bar{v}_0$
α_0
$\bar{v}_{0x}$
Позиція А.
$\bar{v}_y$
$\bar{v}$
α
$\bar{v}_x$
Позиція В.
$m\bar{g}$
Позиція С.
$\bar{v}_x$
α
$\bar{v}_y$
$\bar{v}$
Позиція D.
x_0
x_k
x

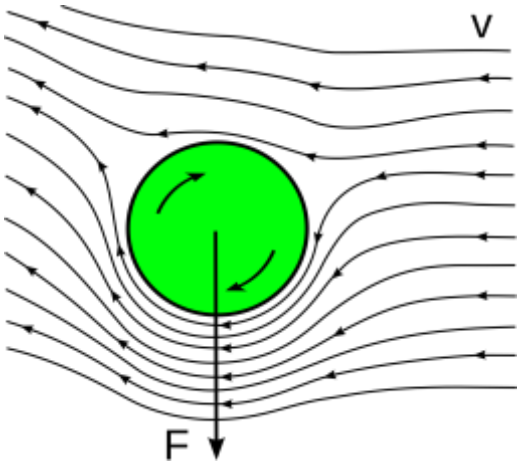

y
y_k
y_0
x_0
x_k
x
$\bar{v}_{0y}$
$\bar{v}_0$
α_0
$\bar{v}_{0x}$
Позиція A.
$\bar{v}_y$
$\bar{v}$
α
$\bar{v}_x$
Позиція B.
$m\bar{g}$
Позиція C.
$\bar{v}_x$
α
$\bar{v}_y$
$\bar{v}$
Позиція D.

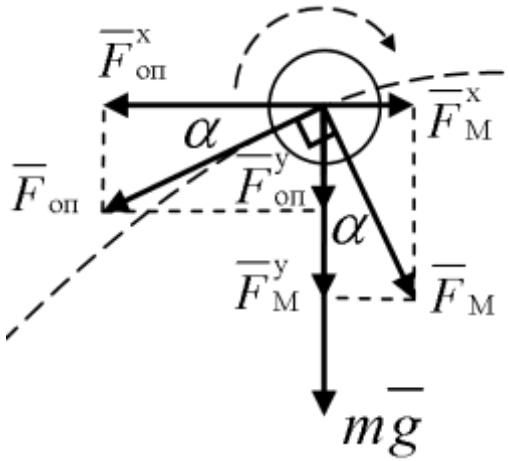


$$\bar{F} = \bar{F}_{\text{т}} = m\bar{g}$$

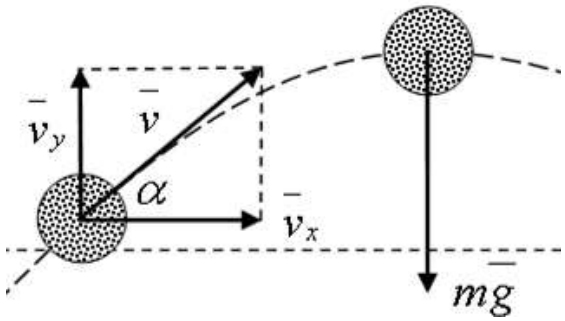


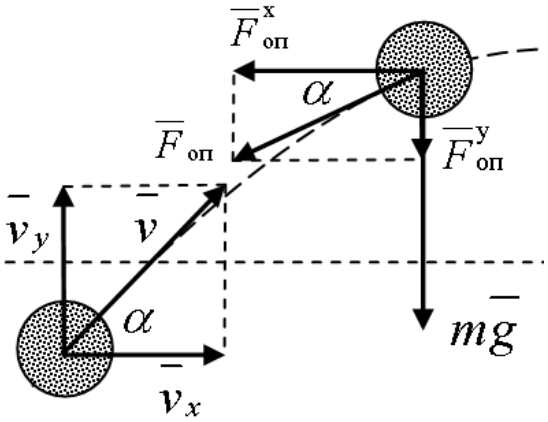
$$\bar{F} = \bar{F}_{\text{т}} + \bar{F}_{\text{оп}}$$

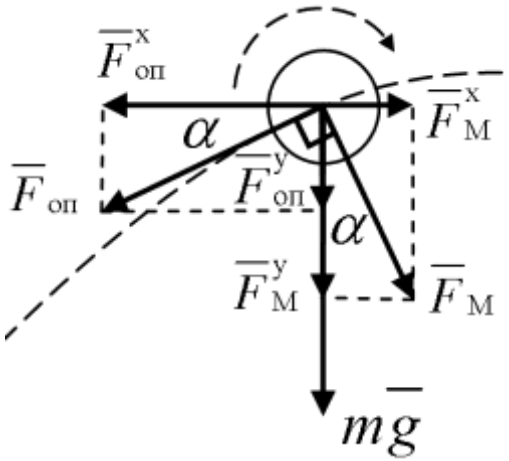




$$\bar{F} = \bar{F}_{\text{т}} + \bar{F}_{\text{оп}} + \bar{F}_{M}$$



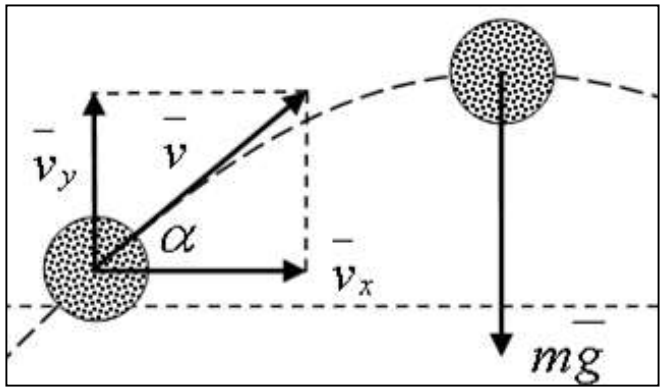




$$\begin{cases} \dfrac{dx}{dt} = v_x \\ \dfrac{dy}{dt} = v_y \\ \dfrac{dv_x}{dt} = 0 \\ \dfrac{dv_y}{dt} = -g \end{cases}$$

$$\begin{cases} \dfrac{dx}{dt} = v_x \\ \dfrac{dy}{dt} = v_y \\ \dfrac{dv_x}{dt} = -\dfrac{k_2}{m} v_x \sqrt{v_x^2 + v_y^2} \\ \dfrac{dv_y}{dt} = -\dfrac{k_2}{m} v_y \sqrt{v_x^2 + v_y^2} - g \end{cases}$$

$$\begin{cases} \dfrac{dx}{dt} = v_x \\ \dfrac{dy}{dt} = v_y \\ \dfrac{dv_x}{dt} = -\dfrac{k_2}{m} v_x \sqrt{v_x^2 + v_y^2} \pm \dfrac{k_3}{m} v_y \sqrt{v_x^2 + v_y^2} \\ \dfrac{dv_y}{dt} = -\dfrac{k_2}{m} v_y \sqrt{v_x^2 + v_y^2} \mp \dfrac{k_3}{m} v_x \sqrt{v_x^2 + v_y^2} - g \end{cases}$$



Math Model

Before collision

$$\frac{dx}{dt} = v_x\,,\ \frac{dy}{dt} = v_y\,,\ \frac{dv_x}{dt} = 0\,,\ \frac{dv_y}{dt} = -g\,, \qquad (1)$$

$x(0) = x_0\,, y(0) = y_0\,,\ v(0) = v_0\,,\ \alpha(0) = \alpha_0\,,$

$x = x(t)\,,\ y = y(t)$ – coordinates of body centre;

$\left(v_x(t), v_y(t)\right) = v(t) = v$ – instant body velocity;

$\alpha = \alpha(t)$ – angle of inclination the body trajectory to the horizon;

$$v_x(t) = v\cos\alpha = v_x(v,\alpha)\,,\ v_y(t) = v\sin\alpha = v_y(v,\alpha)\,,\ v = \sqrt{v_x^2 + v_y^2}\,,\ \alpha = arctg\,\frac{v_y}{v_x}\,.$$

$$x(t) = x_0 + v_{0x}t\,,\ y(t) = y_0 + v_{0y}t - \frac{gt^2}{2}\,,\ v_x = v_{0x}\,,\ v_y = v_{0y} - gt \qquad (2)$$

$$y(x) = y_0 + \frac{v_{0y}}{v_{0x}}(x - x_0) - \frac{g}{2v_{0x}^2}(x - x_0)^2 \qquad (3)$$

Math Model

Collision and after collision

Collision

$$D\left(\frac{x_1+\lambda x_2}{1+\lambda},\frac{y_1+\lambda y_2}{1+\lambda}\right),\ \lambda=\frac{r_1}{r_2},\ r_1 \text{ and } r_2 \text{ – radii of bodies.}$$

$$t_D\left(x_{01},y_{01},x_{02},y_{21},v_{01},v_{02},\alpha_{01},\alpha_{02},r_1,r_2\right)=\left.-b+\sqrt{b^2-4ac}\right/2a \qquad (5)$$

$$a=\left(v_{0x2}-v_{0x1}\right)^2+\left(v_{0y2}-v_{0y1}\right)^2,\ b=2\left(\left(y_{02}-y_{01}\right)\left(v_{0y2}-v_{0y1}\right)+\left(x_{02}-x_{01}\right)\left(v_{0x2}-v_{0x1}\right)\right),$$

$$c=\left(x_{02}-x_{01}\right)^2+\left(y_{02}-y_{01}\right)^2-\left(r_1+r_2\right)^2.$$

After collision

$x_{0i}=x_{iD}$, $y_{0i}=y_{iD}$ – new initial coordinates and velocities:

$$u_{01x}=\frac{\left(m_1-m_2\right)v_{1xD}+2m_2v_{2xD}}{m_1+m_2},\ u_{01y}=\frac{\left(m_1-m_2\right)v_{1yD}+2m_2v_{2yD}}{m_1+m_2};$$

$$u_{02x}=\frac{\left(m_2-m_1\right)v_{2xD}+2m_1v_{1xD}}{m_1+m_2},\ u_{02y}=\frac{\left(m_2-m_1\right)v_{2yD}+2m_1v_{1yD}}{m_1+m_2} \qquad (6)$$

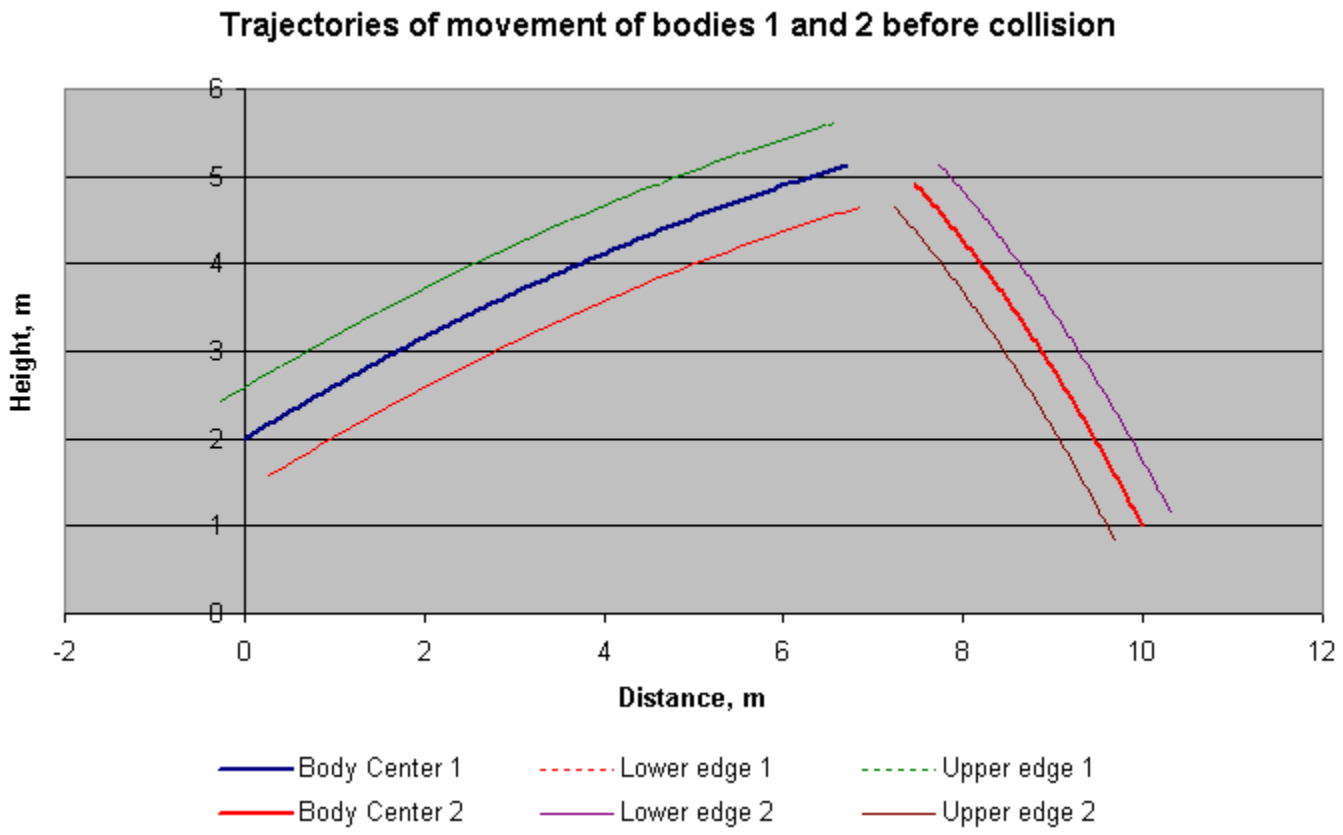
With Excel (Groups E&EG)

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Simulation of the collision of two bodies**								**Time of collision of bodies**			
2									0,4653 s			
3	***General parameters of the model***											
4	**g=**	9,81	m/s2	**n=**	100		t0=	0	s			
5	**tM=**	0,4653	s	**deltaT=**	0,00465	s						
6												
7	**Body 1. Initial data**						**Body 2. Initial data**					
8	**d1=**	1	m				**d2=**	0,7	m			
9	**m1=**	2	kg				**m2=**	1	kg			
10	**x01=**	0	m	v01x=	14,4168	m/s	**x02=**	10	m	v02x=	-5,44789	m/s
11	**y01=**	2	m	v01y=	9,00863	m/s	**y02=**	1	m	v02y=	10,6921	m/s
12	**v01=**	17	m/s				**v02=**	12	m/s			
13	**alpha01=**	32	degrees	0,55851	radians		**alpha02=**	117	degrees	2,04204	radians	

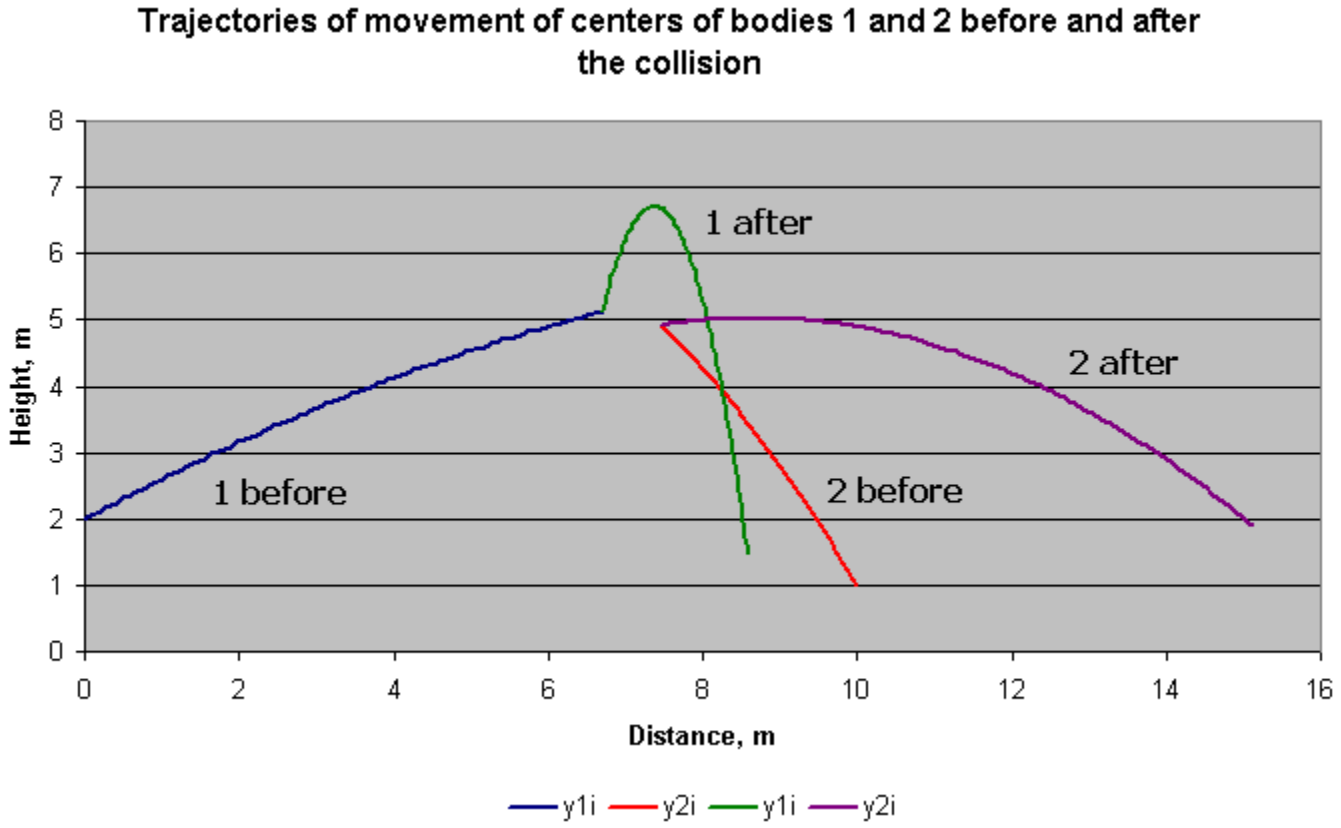
$$n = 100\,,\ \Delta t = \frac{t_M}{n}\,,\ t_i = t_0 + i \cdot \Delta t\,,\ x_i = x_0 + v_{0x} t_i\,,\ y_i = y_0 + v_{0y} t_i - \frac{g t_i^2}{2}\,,\ i = \overline{0,\, n}\,.$$

	A	B	C	D	E	F	G	H	I	J	K	L
16	**i**	**ti**	**x1i**	**y1i**	**x1i-**	**y1i-**	**x1i+**	**y1i+**	**v1xi**	**v1yi**	**v1i**	**alpha1i, рад**
17	0	0	0	2	0,26496	1,575976	-0,26496	2,424024	14,41682	9,008627	17	0,558505361
18	1	0,004653	0,067081	2,041811	0,331073	1,617184	-0,19691	2,466438	14,41682	8,962982	16,97586	0,556225065
19	2	0,009306	0,134163	2,08341	0,397183	1,65818	-0,12886	2,508639	14,41682	8,917336	16,9518	0,553938285
20	3	0,013959	0,201244	2,124796	0,463289	1,698964	-0,0608	2,550627	14,41682	8,87169	16,92783	0,551645017
21	4	0,018612	0,268326	2,165969	0,52939	1,739536	0,007261	2,592403	14,41682	8,826044	16,90395	0,549345259
22	5	0,023265	0,335407	2,206931	0,595488	1,779897	0,075327	2,633965	14,41682	8,780398	16,88017	0,547039007
23	6	0,027918	0,402489	2,24768	0,661581	1,820045	0,143397	2,675314	14,41682	8,734752	16,85647	0,544726258
24	7	0,032571	0,46957	2,288216	0,72767	1,859982	0,211471	2,716451	14,41682	8,689106	16,83286	0,542407009
25	8	0,037224	0,536652	2,328541	0,793754	1,899707	0,279549	2,757374	14,41682	8,64346	16,80934	0,540081258
26	9	0,041877	0,603733	2,368652	0,859835	1,939221	0,347631	2,798084	14,41682	8,597814	16,78592	0,537749003
27	10	0,04653	0,670815	2,408552	0,925911	1,978522	0,415718	2,838582	14,41682	8,552168	16,76258	0,535410242
28	11	0,051183	0,737896	2,448239	0,991984	2,017612	0,483808	2,878866	14,41682	8,506522	16,73934	0,533064974
29	12	0,055836	0,804977	2,487714	1,058052	2,05649	0,551903	2,918937	14,41682	8,460876	16,71619	0,530713197
30	13	0,060489	0,872059	2,526976	1,124116	2,095157	0,620002	2,958795	14,41682	8,41523	16,69313	0,52835491
31	14	0,065142	0,93914	2,566026	1,190175	2,133612	0,688106	2,998439	14,41682	8,369584	16,67017	0,525990112
32	15	0,069795	1,006222	2,604863	1,25623	2,171856	0,756213	3,037871	14,41682	8,323939	16,6473	0,523618804

With Excel (Groups E&EG)



With Excel (Groups E&EG)



The implementation of Math Model by means of GeoGebra … (fragment)

$O_{01}(x_{01}, y_{01})$, $O_{02}(x_{02}, y_{02})$ – starting points of trajectories of bodies 1 and 2;

Vectors of initial velocity:

$\vec{v}_{01x} = \overrightarrow{O_{01}A_{01}}$, $\vec{v}_{01y} = \overrightarrow{O_{01}B_{01}}$, $\vec{v}_{01} = \overrightarrow{O_{01}C_{01}}$; $\vec{v}_{02x} = \overrightarrow{O_{02}A_{02}}$, $\vec{v}_{02y} = \overrightarrow{O_{02}B_{02}}$, $\vec{v}_{02} = \overrightarrow{O_{02}C_{02}}$, where

$C_{01}\left(x(O_{01}) + v_{01x} \cdot mst, y(O_{01}) + v_{01y} \cdot mst\right)$, $C_{02}\left(x(O_{02}) + v_{02x} \cdot mst, y(O_{02}) + v_{02y} \cdot mst\right)$,

$A_{01}\left(x(C_{01}), y(O_{01})\right)$, $A_{02}\left(x(C_{02}), y(O_{02})\right)$, $B_{01}\left(x(O_{01}), y(C_{01})\right)$, $B_{02}\left(x(O_{02}), y(C_{02})\right)$,

$O_1(x_1, y_1)$, $O_2(x_2, y_2)$ – current coordinates the canters of bodies 1 and 2;

vectors of instant velocity:

$\vec{v}_{1x} = \overrightarrow{O_1A_1}$, $\vec{v}_{1y} = \overrightarrow{O_1B_1}$, $\vec{v}_1 = \overrightarrow{O_1C_1}$; $\vec{v}_{2x} = \overrightarrow{O_2A_2}$, $\vec{v}_{2y} = \overrightarrow{O_2B_2}$, $\vec{v}_2 = \overrightarrow{O_2C_2}$, where

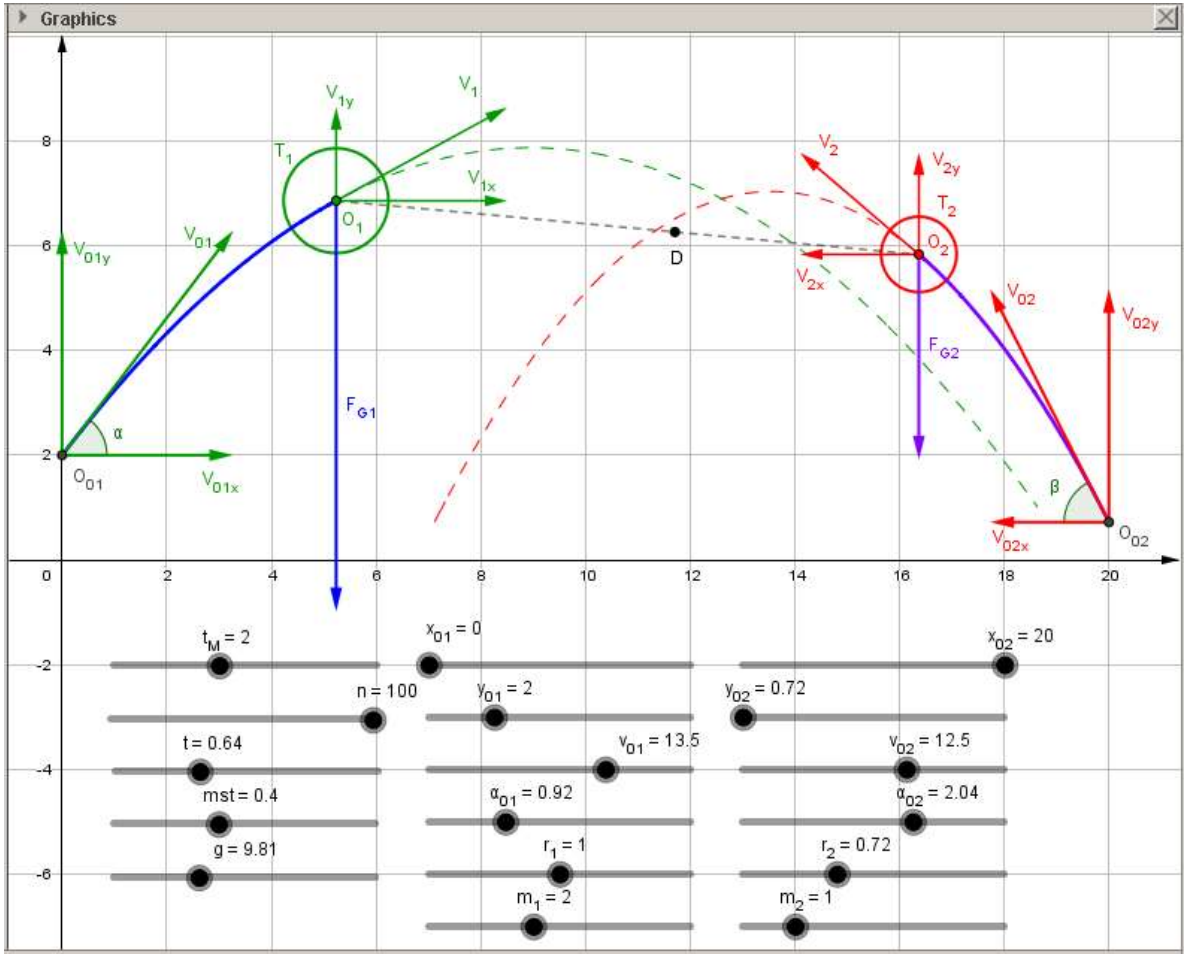
$C_1\left(x(O_1) + v_{1x} \cdot mst, y(O_1) + v_{1y} \cdot mst\right)$, $C_2\left(x(O_2) + v_{2x} \cdot mst, y(O_2) + v_{2y} \cdot mst\right)$,

$A_1\left(x(C_1), y(O_1)\right)$, $A_2\left(x(C_2), y(O_2)\right)$, $B_1\left(x(O_1), y(C_1)\right)$, $B_2\left(x(O_2), y(C_2)\right)$.

Vectors of forces acting on bodies 1 and 2: $\overrightarrow{F_{G1}} = \overrightarrow{O_1F_1}$, $\overrightarrow{F_{G2}} = \overrightarrow{O_2F_2}$, where

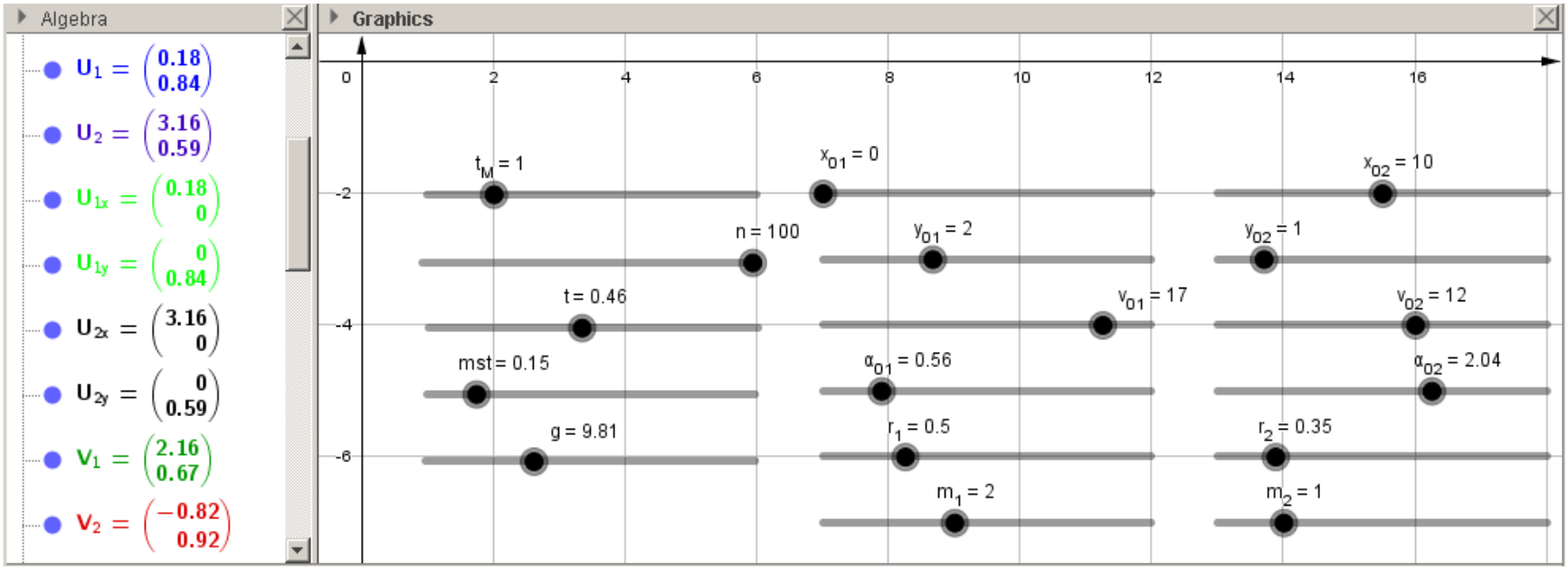
$F_1 = \left(x(O_1), y(O_1) - m_1 \cdot g \cdot mst\right)$, $F_2 = \left(x(O_2), y(O_2) - m_2 \cdot g \cdot mst\right)$.

With GeoGebra (Group G&EG)



With GeoGebra (Group G&EG) collision moment visualization

With GeoGebra (Group G&EG) collision moment parameters



Results

The ETF PDAA students' learning outcomes at 2018

Methodology for assessing the performance results of an individual independent learning tasks by the students

Final score	Bal	The achiev-ing level	The correctness task implementation	Self-support the perform-ing of the learning task
A	90-100	The task is full complete	without remarks and errors	by yourself
B	82-89	The task is full complete	with minor comments and / or inaccuracies that did not affect the result	by yourself
C	74-81	The task is full complete	with remarks and / or inaccuracies corrected by the student	by yourself, with little help from the teacher
D	64-73	The task is not full com-plete	with remarks and / or inaccuracies that affected the result and were completely corrected by the student	with help from the teacher
E	60-63	The task is not full com-plete	with significant comments and / or inaccuracies affecting the result and were partially corrected by the student	with help from the teacher
FX	35-59	The task is uncomplete	There were errors that affected the result and were only partially corrected by the student	with help from the teacher
F	0-34	The task is uncomplete	There were errors that affected the result and were not corrected by the student	with help from the teacher

Final learning outcomes of students after the experiment

Table 2. Final learning outcomes of students after the experiment.

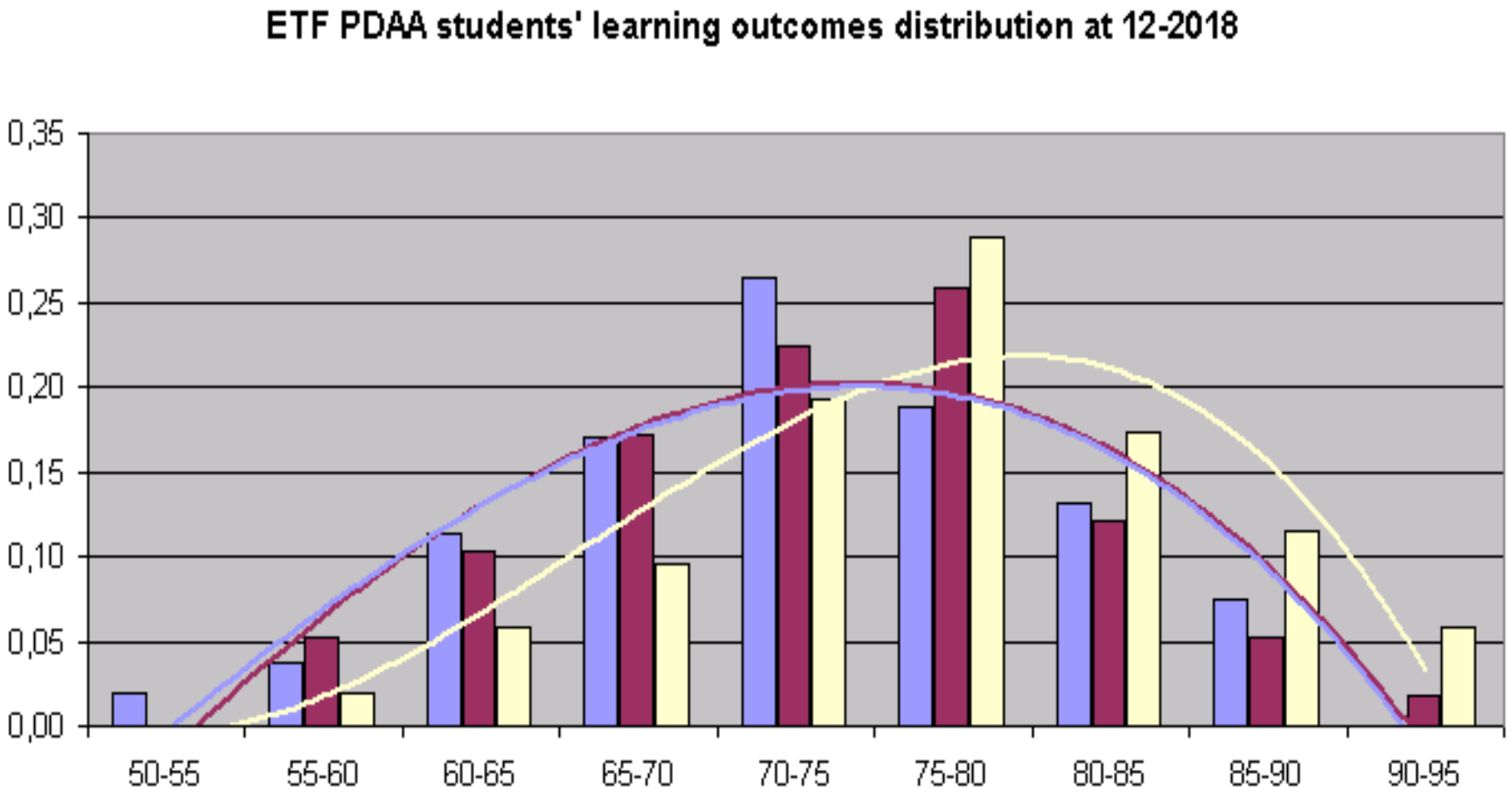
Bal	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95
Group E	1	2	6	9	14	10	7	4	-
Group G	-	3	6	10	13	15	7	3	1
Group EG	-	1	3	5	10	15	9	6	3

Table 3. The primary statistical data processing results.

Group	Valid N	Mean	Conf. -95%	Conf. +95%	Median
E	53	73,4	71,2	75,6	74
G	58	73,7	71,6	75,8	74
EG	52	77,4	75,7	79,6	77

Group	Mode	Frequency Mode	Min	Max	SD	Shapiro-Wilk test	
E	Multiple		55	90	7,95	W=0,98823	p=0,87840
G	76	5	56	91	8,06	W=0,98859	p=0,86117
EG	76	6	60	95	7,96	W=0,98956	p=0,92711

ETF PDAA students' learning outcomes distribution (percentage valid) at 2018



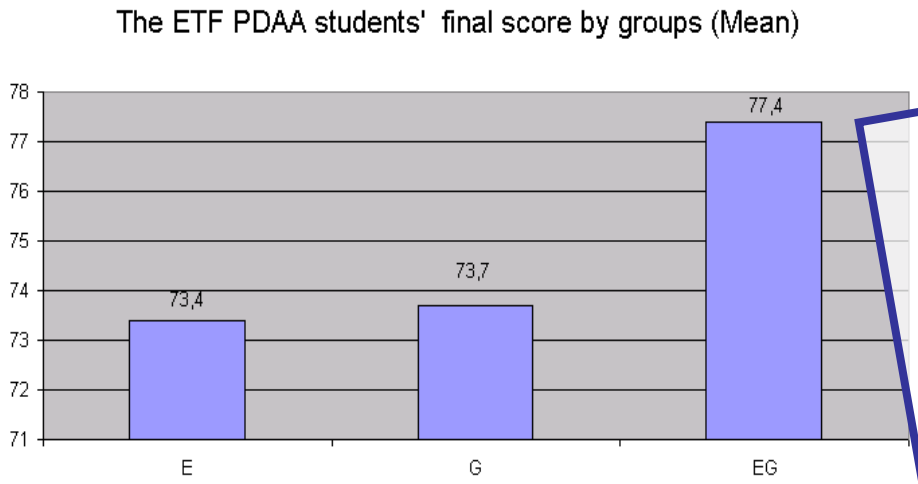
The ANOVA results

Table 4. The ANOVA results.

	SS	df	MS	SS	df	MS	F	p
Bal	528,7585	2	264,3793	10226,91	160	63,91819	4,136213	0,017724

Table 5. The pair-wise post-hoc comparisons results.

Pair-wise post-hoc comparisons of means	E vs. G	E vs. EG	G vs. EG
LSD-test	p>0,8277	p<0,0108	p<0,0166
Duncan-test	p>0,8293	p<0,0120	p<0,0161
Tukey HSD for unequal N	p>0,9753	p<0,0276	p<0,0161



The difference of mean by groups E,G vs EG is significant

Thanks for your attention!!!

THE SIMULTANEOUS USE OF EXCEL AND GEOGEBRA TO TRAINING THE BASICS OF MATHEMATICAL MODELING

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