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Exact Solution of the Problem of Elastic Bending of a Multilayer Beam under the Action of a Normal Uniform Load

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Abstract. An exact solution of the theory of elasticity is presented for the problem of a narrow multilayer bar section transverse bending under the action of a normal uniform load on longitudinal faces. The solution is built using the principle of superposition, by imposing common solutions to the problems of bending a multilayer cantilever with uniform loads on the longitudinal faces and an arbitrary load on the free end, and allows to take into account the orthotropy of the materials of the layers, as well as transverse shear deformation and compression. On the basis of a built-in general solution, a number of particular solutions are obtained for multi-layer beams with various ways of the ends fixing.

Introduction

Beams that work on a flat transverse bend under the action of a uniform load on the longitudinal faces, are quite common elements of various engineering structures. Therefore, the definition of the stress-strain state (SSS) of such elements is of great practical importance and is one of the classical problems of mechanics of a deformed solid.

The exact solutions of the theory of elasticity for individual problems of bending of isotropic beams under the action of a uniform load have been known for a long time. As a special case of solving the plane problem of the theory of elasticity in polynomials, Timpe in [1] obtained a solution for an isotropic console with a uniform load along the length. Built on the basis of this solution, the solution for a simply supported beam is a classic problem of the theory of elasticity course [2]. For a more complicated case of rigid supports of the ends, as well as a combination of rigid and hinged supports, solutions were obtained relatively recently in [3, 4].

Similarly, for an anisotropic and, in particular, an orthotropic console, the solution to the problem of bending under a uniform load was also obtained quite a long time ago [5]. However, solutions for beams with different end supports were obtained much later in [6, 7].

In a number of recent works [6-12], the efforts of scientists were directed to solving the problems of bending anisotropic functionally gradient beams, the material of which has continuous non-uniformity over the height of the section.

At the same time, the problems of multilayer beams bending remain practically unexplored, although the composite elements used in practice are predominantly multilayered. The exact solution of the theory of elasticity was obtained only for a three-layer beam with different types of end supports [13]. But, the layer-by-layer approach used in this paper to solving the problem leads to very cumbersome relations, even for a beam with isotropic layers.

However, for multilayer beams, refined bending models are quite developed, for example, [14-17].

In [18, 19], the authors proposed solutions for multilayer cantilevers with a load on the free end and a uniform load on the longitudinal faces, which were obtained using a continuous approach to the description of a multilayer structure. Such an approach makes it possible to obtain uniform relations for the characteristics of the SSS at once for the whole package of layers, which may contain isotropic or orthotropic layers, homogeneous or continuously heterogeneous with an

Similarly to the previous examples, for comparison with known ratios, we give the resulting ratio for the deflection of the lower fiber in the middle section. For the case of conditions Eq. 26 and Eq. 29, this value will be the same and equal to

$$w|_{x=\frac{1}{2}l,z=z_1} = -\frac{p^{\Sigma}l^4}{384B_2}\left(1-\left[\frac{48D_2}{hl^2}\right]\right). \tag{31}$$

For homogeneous orthotropic and isotropic beams with fixed ends, the relation Eq. 31, taking into account Eq. 15, is simplified to the form

$$w|_{x=\frac{1}{2}l,z=z_1} = \frac{p^{\Sigma}bl^4}{384E_xJ_y}\left(1+\left[4\left(\frac{E_x}{G_{xz}}-\nu_{xz}\right)\frac{h^2}{l^2}\right]\right),\ w|_{x=\frac{1}{2}l,z=z_1} = \frac{p^{\Sigma}bl^4}{384E_xJ_y}\left(1+\left[4(2+\nu)\frac{h^2}{l^2}\right]\right). \tag{32}$$

In square brackets in relations Eq. 31 and Eq. 32 a component is selected that takes into account the lateral shear and compression deformations, and is a refinement of the expression for the deflection rise of a rigidly fixed beam according to the elementary bending theory.

It should be noted that such a refinement significantly affects the amount of deflection than in the examples discussed above. So for a carbon fiber rectangular beam with a ratio of $l/h=10$, the value of the specified specification will be 102.8%, and at $l/h=5$ – 411.1%. The FEM simulation for a rigidly fixed carbon fiber beam at $l/h=5$ gives a deflection value 15.6% greater than the first relation Eq. 32, and by 9.6% more with $l/h=10$.

Summary

Thus, an exact analytical solution has been constructed for the problem of plane elastic bending of a section of a multilayer beam under the action of a uniform normal load on longitudinal faces, which compose relations Eq. 2-4. The solution obtained makes it possible to determine the components of the SSS of multilayer bar consisting of an arbitrary number of orthotropic homogeneous or continuously inhomogeneous layers, taking into account the compliance of their materials to transverse shear deformations and compression.

The application of the obtained solution together with the proposed analogue of the method of initial parameters made it possible to obtain a number of new solutions for problems of bending of beams with different types of the ends supports.

Comparison of the calculation results of the deflections of beams using the obtained relations, with the FEM simulation results, show only a slight difference even for short beams. In this case, the proposed methods for modeling fixings are not the only possible ones, and the list of methods for supports can be significantly expanded.

The obtained general solution for the multi-section beam section and the proposed method for determining the initial parameters can be generalized and extended to the case of multi-span beams with an arbitrary number of sections, with different load intensity and structure.

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