

**ELASTICITY THEORY SOLUTION OF THE PROBLEM  
ON BENDING OF A NARROW MULTILAYER CANTILEVER  
WITH A CIRCULAR AXIS BY LOADS AT ITS END**

**S. B. Koval'chuk\* and A. V. Goryk**

**Keywords:** *curved bar, circular axis, composite, layer, bending, stress, strain, displacement*

*An exact solution of the problem on plane bending of a narrow multilayer cantilever bar with a curved circular axis by tangential and normal loads distributed on its free end face is presented. The natural (for a bar structure) cylindrical circular coordinate system is used to describe the structure and geometry of the bar. The solution is obtained by directly integrating the equations of a plane elasticity theory problem using an analytical description of the mechanical characteristics of the discrete-inhomogeneous multilayer bar. Physical relations that take into account the cylindrical orthotropy of the material of bar layers and the conditions of absolutely rigid contact of layers are used in constructing the solution. The theoretical relations are realized for the test problem on determining the strain-stress state of a four-layer cantilever with a semiring axis. The solution obtained allows one to predict the strength and rigidity, to develop optimum design techniques, and to construct analytical solutions of different problems on bending of multilayer curved bars.*

**Introduction**

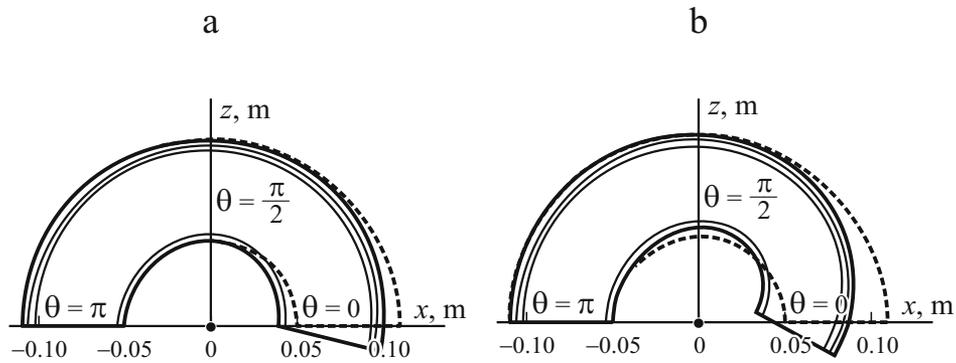
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Poltava State Agrarian Academy, Poltava, Ukraine  
\*Corresponding author; e-mail: stanislav.kovalchuk@pdaa.edu.ua

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