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ANALYTICAL SOLUTION OF THE PROBLEM OF THERMOELASTIC DEFORMATION OF A NONUNIFORMLY ROTATING MULTILAYER DISK

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Explicit analytical solution to the problem of the strain–stress state of a narrow multilayer disk with a radial alternation of layers is presented. The disk rotates with acceleration, in an axisymmetric temperature field, under the action of normal and tangential loads uniformly distributed on its cylindrical surfaces. The relations are obtained by solving the system of equations of the plane problem of elasticity in a polar coordinate system, given a discretely inhomogeneous structure of the disk. The solution obtained gives the distribution of stresses and displacements throughout the disk layers and can be used to address a wide range of applied problems and methods of optimal design.

Keywords: composite disk, concentric layers, rotation, acceleration, temperature field, stress, strain, displacement

Introduction and Problem Statement. Commonly, the mechanisms of power machines have parts in the form of circular disks rotating with a large angular velocity and acceleration in a nonuniform temperature field. They can perform auxiliary functions, like flywheels, or be load-bearing elements like wheels of mechanical transmissions, disks of working bodies of grinders, axial turbine wheels, etc. Compressor and turbine disks are one of the most important parts of aircraft gas turbine engines, as the perfection of their design largely determines their reliability, as well as technical and economic characteristics, such as strength and minimum weight.

Improving the weight characteristics of gas turbine engine disks is possible, in particular, by using composite materials for their manufacture [2, 4, 7], as well as by alternating layers of different materials in the radial direction. Designing such disks with optimal parameters requires reliable models of their stress–strain state (SSS). Finite-element modeling is quite universal and indispensable at the stage of verification of composite disk design. At the same time, at the stage of preliminary calculations, analytical models that make it possible to analyze dependences of the SSS components on various properties of materials and parameters of external effects are useful. However, in terms of discretely inhomogeneous multilayer disks under the action of external loads of different nature, they are still underdeveloped.

In most research works dealing with the theoretical study of the SSS of rotating isotropic and cylindrically orthotropic disks, their material is considered perfectly elastic. A separate area of research in this field is studying the influence of plasticity and creep on the disk SSS. At the same time, fibrous composite materials while stretching/compressing along the fibers show a linear relationship between stresses and strains almost to rupture [1], which allows the use of analytical solutions based on the elasticity theory.

In [8], a solution for rotating thin isotropic disks of constant and variable thickness with or without a hole was considered. It was obtained using the assumption of a flat stress state. This assumption is justified not only by a significant simplification of the initial equations, but also by a small discrepancy in the results in comparison with the well-known spatial

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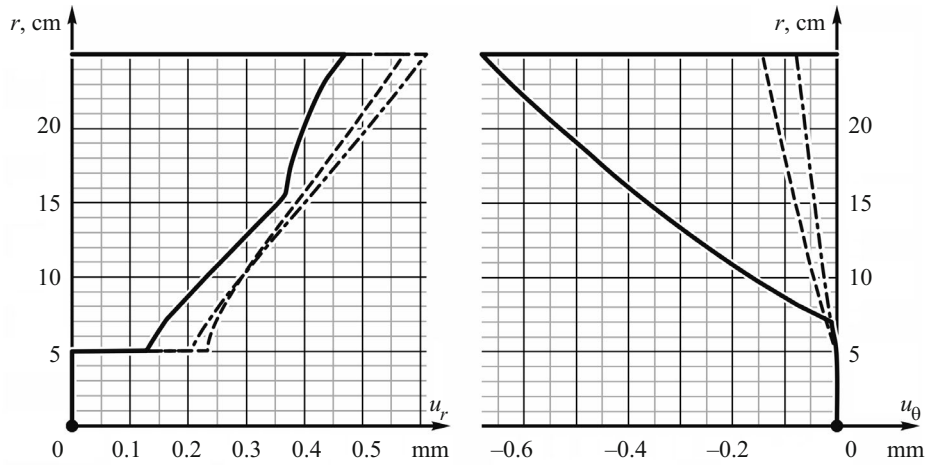

a *b*

Fig. 5

The magnitude of the tangential stresses mainly depends on the tangential load on the periphery of the disk, so their share in the bulk forces of inertia during accelerated rotation is relatively small. At the same time, the obtained results do not exclude the fact that in the case of other initial data, the forces of inertia can significantly affect the magnitude of the tangential stresses.

The stiffness of the composite disk is higher than that of the solid titanium and steel disks, in the radial direction, but lower, in the circumferential direction.

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