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EXACT SOLUTION OF THE PROBLEM ON ELASTIC BENDING OF THE SEGMENT OF A NARROW MULTILAYER BEAM BY AN ARBITRARY NORMAL LOAD

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An exact analytical solution of the problem on plane elastic bending of the segment of a narrow multilayer beam under the action of arbitrary normal loads distributed over its longitudinal faces is presented. It is assumed that the beam deforms elastically, its layers are made of orthotropic materials homogeneous or continuously heterogeneous across the thickness of layers, which are rigidly connected together, and the load is given as the sum of a trigonometric series. This allowed us to reduce the solution of the given problem on bending to the solution of the auxiliary problem on bending of a multilayer cantilever beam under the action of a sinusoidal load with an arbitrary number of half-waves. Its solution is obtained solving the equations of plane elasticity by using an analytical description for the variables of mechanical characteristics of the multilayer structure. The solution of the original problem is found as the sum of general solutions of the problems for the multilayer cantilever with a load at its free end and with sinusoidal loads on the longitudinal faces. The theoretical relations obtained are checked by solving the test problem on bending of a five-layer hinged beam with a linear discontinuous load. To reduce the Gibbs effect in the vicinity of jump discontinuities of load on its approximation by the partial sum of a trigonometric series, the Lancos method was used. The results obtained are confirmed by the results of a finite-element modeling. The solution constructed enables one to take into account an arbitrary distribution of normal load on longitudinal beam faces, including local loads and loads on a surface section, and can be used for predicting the strength and stiffness of multilayer beams and, with small changes — for solving contact problems for such structural elements.

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